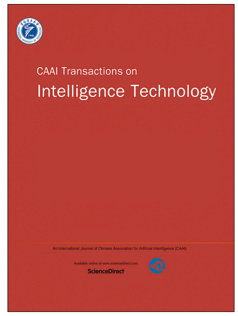

A Multi-objective Optimization Framework for Ill-posed Inverse Problems in Image Processing

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A Multi-objective Optimization Framework for Ill-posed Inverse Problems in Image Processing

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***Abstract*—Many image inverse problems are ill-posed for no unique solutions. Most of them have incommensurable or mixed-type objectives. In this study, a multi-objective optimization framework is introduced to model such ill-posed inverse problems. The conflicting objectives are designed according to the properties of ill-posedness and certain techniques. Multi-objective evolutionary algorithms have capability to optimize multiple objectives simultaneously and obtain a set of trade-off solutions. For that reason, we use multi-objective evolutionary algorithms to keep the trade-off between these objectives for image ill-posed problems. Two case studies of sparse reconstruction and change detection are implemented. In the case study of sparse reconstruction, the measurement error term and the sparsity term are optimized by multi-objective evolutionary algorithms, which aims at balancing the trade-off between enforcing sparsity and reducing measurement error. In the case study of image change detection, two conflicting objectives are constructed to keep the trade-off between robustness to noise and preserving the image details. Experimental results of the two case studies confirm the multi-objective optimization framework for ill-posed inverse problems in image processing is effective.**

***Index Terms*—Ill-posed problem, image processing, multi-objective optimization, evolutionary algorithm.**

I. Introduction

ILL-POSED problems were proposed by Hadamard in the beginning of last century [1]. A problem is defined to be ill-posed if there is no unique solution. Hadamard first defined a mathematical problem to be well-posed when its solution (i) exists; (ii) is unique and (iii) depends continuously on the initial data; otherwise, a problem is ill-posed. An arbitrary small perturbation of the data can cause an arbitrary large perturbation of the solution. Most of the classical physics problems are well-posed. However, their inverse problems are usually ill-posed. Today there is a vast amount of literature on ill-posed problems arising in many areas of science and engineering [2]–[5].

Many image processing problems are ill-posed in the sense of Hadamard, such as denoising, deblurring, inpainting and so on. Most of ill-posed problems are not sufficiently constrained. In recent decades, a lot of generic constraints on the problem were introduced to regularize them and make them well-posed. One of the best way to "cure" ill-posed problems is to transform them to well-posed ones by adding (one or more) regularization terms [6]. The regularization methods construct approximate solutions of ill-posed problems that are stable under small changes in the initial data. The problems of finding an approximate solution in regularization methods are to find a appropriate regularizing operator and to determine the regularization parameter α from supplementary information pertaining to the problem. Tikhonov regularization is one of the most common and well-known form of regularization methods [6]. The penalty term in Tikhonov function takes into account the additional information about the solution and its role is to stabilize the problem and to single out a useful and stable solution. Besides Tikhonov regularization, there are many other regularization methods with properties that make them better suited to certain ill-posed problems [7]–[9]. For example, the total variation (TV) model has been introduced by Rudin-Osher and Fatemi (ROF) in [7] as a regularization method. TV operator has been used extensively and with great success for inverse problems because it is able to smooth noise in flat areas of the image. Furthermore, it is a nontrivial application-dependent task to choose suitable parameter values. A large α favors a small solution seminorm at the cost of a large residual norm, while a small α has the opposite effect. The choice of the admissible value of parameter depends on the information available with respect to the approximate initial information. Many quantitative parameter optimization methods have been proposed in recent decades, such as the L-curve method, generalized cross-validation, the discrepancy principle and estimation of mean-squared error.

In many cases, the solution to ill-posed problems can be forced to be unique by narrowing the solution space and adding additional information. Another considerable way is to find multiple trade-off solutions and then choose one or more suitable solutions as required. From this respect, it seems that image ill-posed problems can be modeled as problems with incommensurable or mixed-type multiple objectives. Multi-objective evolutionary algorithms (MOEAs) are able to simultaneously optimize multiple objectives to keep the trade-off between these objectives and generate a set of trade-off solutions in a single run [10], [11]. For this reason, MOEAs are suitable to solve this kind of problems. The decision makers can judge relatively and select one or more suitable solutions according to the problem requirements.

In this paper, a multi-objective optimization (MO) framework is introduced to solve ill-posed inverse problems in image processing. Sparse reconstruction and change detection are used as two case studies to show how the MO framework can be successfully used to solve image ill-posed problems.

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