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Logical composition of qualitative shapes applied to solve spatial reasoning tests

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Abstract

A logical approach to compose qualitative shape descriptors (LogC-QSD) is presented in this paper. Each object shape is described qualitatively by its edges, angles, convexities, and lengths. LogC-QSD describes the shape of composed objects qualitatively adding circuits to describe the connections among the shapes. It also infers new angles and lengths using composition tables. Its main contributions are: (i) describing qualitatively the resulting boundary of connecting N shapes and (ii) its application to solve spatial reasoning tests. LogC-QSD approach has been implemented using Prolog programming language, which is based on Horn clauses and first order logic. The testing framework was SWI-Prolog on the LogC-QSD dataset. The obtained results show that the LogC-QSD approach was able to correctly answer all the questions in the LogC-QSD dataset, which involved compositions up to five shapes. The correct answer for 60% of the questions was obtained in an average time of $2.45 \cdot 10^{-4}$ s by comparing the concavities and right angles of the final QSD composed shape with the possible answers. The rest of the questions required a matching algorithm and they were solved by LogC-QSD in an average time of $19.50 \cdot 10^{-4}$ s. Analysis of the execution times obtained showed that the algorithmic cost of LogC-QSD is lower than $O(n^2)$ in the worst case.

Keywords: Qualitative spatial reasoning; Qualitative shape descriptor; Shape composition; Angle composition; Length composition; Logic; Tangram; Puzzle; Spatial cognition; Spatial reasoning tests; Prolog; Logic programming

1. Introduction

Spatial cognition studies have shown that there is a strong link between success in Science, Technology, Engineering and Math (STEM) disciplines and spatial skills (Newcombe, 2010; Wai, Lubinksi, & Benbow, 2009). Spatial skills play an important role in many jobs, for example when visualising the planning or result of a surgery, when designing bridges, aircrafts, when interpreting charts, maps, diagrams, engineering drawings, etc.

Qualitative modelling (Forbus, 2011):

> concerns the representations and reasoning that people use to understand continuous aspects of the world. Qualitative models formalise everyday notions of causality and provide accounts of how to ground symbolic-relational representations in perceptual processes.

In the literature, qualitative models have been widely applied in AI, for example, *QIDL*$^+$ extracts logics from digital images (Falomir, 2017; Falomir & Olteţeanu, 2015), the QS-GRI model represents and reasons about interactions between a robot and a group of humans (Falomir & Angulo, 2017), other approaches were applied for robot

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orientation and navigation indoors (Falomir, Museros, Castelló, & Gonzalez-Abril, 2013), in architecture and design (Bhatt & Freksa, 2015), for spatial query solving in geographic information systems (Fogliaroni, 2013), for classification of volunteered geographic information (Ali, Falomir, Schmid, & Freksa, 2017), etc.

In cognitive science, qualitative models have been developed to solve perceptual tests. A qualitative model for describing 3D objects (Q3D) using depth and different perspectives (Falomir, 2015) was defined to help users solving tests where two views of an object are given and the third corresponding view is left. A qualitative descriptor for solving paper folding tests was defined by establishing a correspondence between the possible folding actions and the areas in the paper where a hole can be punched (Falomir, 2016). A logic-based formalisation of the Fisherman's Folly puzzle was proposed applying qualitative spatial reasoning on strings and holes and reasoning about actions and change on them (Cabalar & Santos, 2011). And qualitative descriptors and analogical reasoning were used for solving Raven's Progressive Matrices intelligence test (Lovett & Forbus, 2017).

In the context of creativity, spatial descriptors and qualitative shape and colour descriptors and their similarity formulations were tools for object replacement and object composition in a theoretical approach to solve Alternative Uses Test (Olteteanu & Falomir, 2016).

Traditional algorithms in AI can solve puzzles, although there are some puzzles which require a high computational cost (Brand, 2015). Humans, in contrast, seem to solve puzzles more efficiently. In this paper, we study how spatial logics can improve common reasoning about space. Our main motivation is developing algorithms of spatial reasoning able to solve shape composition tasks, so that intelligent systems (e.g. robots) can enhance their spatial intelligence and apply it to solve a puzzle or to interact with the world more efficiently.

This paper presents LogC-QSD, an approach for describing interconnected objects and their shapes qualitatively. LogC-QSD is inspired by the Qualitative Shape Descriptor (QSD) (Falomir, Gonzalez-Abril, Museros, & Ortega, 2013), the juxtaposition scheme QSD-Jux (Museros, Falomir, Gonzalez-Abril, & Velasco, 2011) and the Point-Line-Circuit-Area (PLCA) framework (Takahashi, Goto, & Miwa, 2015). The Qualitative Shape Descriptor (QSD) (Falomir, Gonzalez-Abril et al., 2013) is used to describe the shape of the initial objects. The LogC-QSD describes the boundary of the final composed object and the interconnections of objects as networks or circuits. In the literature, the QSD-Jux tackled this challenge before (Museros et al., 2011) describing the juxtaposition of two shapes by an edge. The LogC-QSD approach presented in this paper goes a step forward being able to compose N shapes and allowing various kinds of connections (e.g., point-line, point-point, line-point). The Point-Line-Circuit-Area (PLCA) framework (Takahashi et al., 2015) can represent composition of objects and their connections, but the shapes of the objects and the shape of the final composed object cannot be described since the relation between the edges is not stored (e.g., angle, convexity and relative length).

The rest of the paper is organised as follows. Section 2 outlines spatial tests referring to object composition. Section 3 presents the extended Qualitative Shape Descriptor (QSD^*) used to describe the boundaries of the initial objects. Section 4 outlines LogC-QSD, the shape composition operation between objects described by QSD^*'s. Section 5 presents the kind of connections that can happen when composing two shapes. Section 6 explains how the connections between N objects can be described as a network. Section 7 describes the composition tables of the QSD^* features regarding edge-connections, angles and convexities and relative lengths. Section 8 explains how to apply the resulting LogC-QSD to solve spatial reasoning tests using a matching algorithm. Section 9 presents the implementation of LogC-QSD approach and the experimentation carried out. Section 10 compares the LogC-QSD approach with other puzzle solving approaches in the literature. Finally, conclusions and future work are presented.

2. Outlining spatial tests on object composition

Spatial reasoning questions in psychometric tests (i.e. those by Mcmunn (2010a)) involve object composition questions. An example of a test question is showed in Fig. 1. The instructions provided are the following: *Take a look to the presented shapes. Note the letters on the side of each shape. Join these shapes together with the corresponding letters to make the corresponding shape. Look at the given shapes and decide which of the examples matches the final shape built when all the shapes joined together by the corresponding letters. You have 3 min to answer 8 questions.*

Note that the correct answer to the question in Fig. 1 is the option C and any of its rotations (for instance, the hexagon on the top and the triangle on the left).

Fig. 2 presents other kind of spatial reasoning problems (e.g. Tangram puzzle, T-puzzle) that provide the original pieces and the shape of a final composed object using these pieces. The challenge here is to find the right connections.

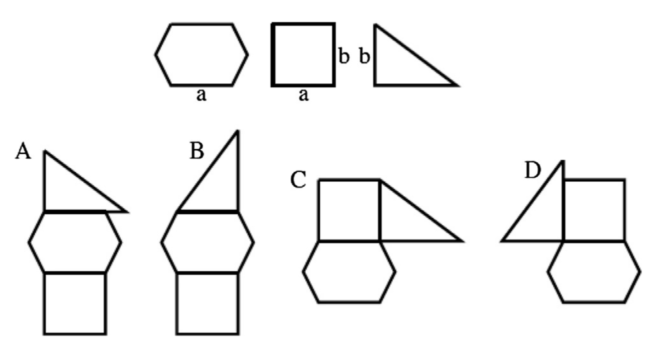

Fig. 1. Drawings reproducing Question 3 at Psychometric Tests by Mcmunn (2010a).