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# A robust correlation coefficient measure of dual hesitant fuzzy soft sets and their application in decision making

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ABSTRACT

The objective of this work is to present novel correlation coefficients for measuring the relationship between two dual hesitant fuzzy soft set (DHFSSs). In the existing studies of fuzzy and intuitionistic fuzzy sets, the uncertainties which are present in the data are handled without considering the parameterizations factor of each expert during the process, which may lose some useful information of alternatives and hence affect the decision results. On the other hand, soft set theory handles the uncertainties by considering both the parameterizations as well as criteria during the evaluation of the object. Thus, motivated by this, we develop correlation coefficient and weighted correlation coefficients under the DHFSS environment in which pairs of membership, non-membership are to be considered as vector representation during the formulation and to investigate their properties. Further, under this environment, a multicriteria decision making method based on the proposed correlation coefficients are presented. Three numerical examples, one from the selection procedure and other from the medical diagnosis and pattern recognition, are taken to demonstrate the efficiency of the proposed approach and compared their results with the several existing approaches results.

## 1. Introduction

Multicriteria decision making (MCDM) is one of the hot topic in these days to select the best alternative(s) from the set of the feasible one. MCDM consists of criteria, alternatives, evaluation values and decision methods in which evaluation values include both precise data and experts' subjective estimations. But due to the complexity of system day-by-day, the real-life contains many MCDM problems in which the information related to the alternatives and the criterion weights are inaccurate, uncertain or incomplete. The best way to deal with such situations is theory of fuzzy set (FS) (Zadeh, 1965) or extended fuzzy sets such as intuitionistic fuzzy set (IFS) (Atanassov, 1986), interval-valued IFS (IVIFS) (Atanassov and Gargov, 1989), Pythagorean fuzzy set (Yager and Abbasov, 2013). Numerable attempts have been made by the different researchers in processing the information values using different operators (Garg, 2018f, 2017c; Wang et al., 2013; Bai, 2013; Garg, 2017d, a; Xu, 2007; Xu and Yager, 2006), information measures (Kumar and Garg, 2016, 2017; Ye, 2010; Wang et al., 2014; Singh, 2017), score and accuracy functions (Garg, 2016a) under these environments. Among all these diverse concepts, one is to find the most suitable alternative making use of correlation coefficients which plays a significant role in statistical and engineering applications as it provides us with measurement of the interdependency of two variables.

In statistical analysis, the correlation coefficients plays a vital role to measure the linear relationship between the two variables, whereas in fuzzy set theory the correlation measure determines the degree of dependency between two fuzzy sets. In that direction, Hung and Wu (2001) firstly defined the correlation coefficient for fuzzy numbers. Gerstenkorn and Manko (1991) introduced the correlation coefficients of IFSs which was later analyzed by Szmidt and Kacprzyk (2010). Bustince and Burillo (1995) presented the correlation coefficient for IVIFS. Garg (2016b) presented the correlation coefficient for Pythagorean fuzzy set. Garg (2018d) presented the correlation coefficients for the intuitionistic multiplicative sets and applied them to solve the problems of pattern recognition and decision making. In these above theories, it is assumed that preferences given by the experts are either a single number or an interval. However, in such situation, these conditions may be unfulfill by an expert. To manage such situations, the concept of the hesitant fuzzy sets (HFSs) was introduced by Torra and Narukawa (2009) and Torra (2010). HFSs are tremendously useful in handling the situations where people have hesitancy in providing their preferences over objects in a decision making process. Chen et al. (2013) discussed the correlation

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coefficients of HFSs and their applications to clustering analysis. Xu and Xia (2011) discussed the correlation measures for HFSs. After their successful applications, Zhu et al. (2012) extended the HFS to dual HFS (DHFS), which comprises of two sets of characteristic functions whose degrees sum is less than one. Under this environment, Ye (2014) and Farhadinia (2014) presented the correlation coefficients and applied them to solve the multi-attribute decision making problems.

The above theories and their corresponding approaches are widely used by the researchers but there approaches insufficiently to consider the parameterization tool during the decision making process. To overcome it, Molodtsov (1999) initiated a soft set (SS) theory which is a new mathematical tool for dealing with the uncertainties. After their existence, many scholars shows their intense interest in it and hence proposed some approaches and its extensions. Maji et al. (2001a) extended the theory of SS to the fuzzy soft set (FSS). Maji et al. (2001b) extended the theory to the intuitionistic fuzzy soft set (IFSS) which is based on the combination of the intuitionistic fuzzy sets and soft set models. Jiang et al. (2010) presented the concept of the interval-valued IFSS by combining SS theory with the interval-valued IFS. Later on, Garg et al. (2016) presented the concept of the fuzzy number IFSS and their basic operational laws such as union, intersection, complement. Since its appearance, it has been widely used by the researchers and many fruitful results have been achieved in the theoretical aspect by combining soft sets with other mathematical structures (Jiang et al., 2010; Maji, 2009; Yang et al., 2009). In the field of the aggregation process, Arora and Garg (2018b) firstly presented some weighted averaging and geometric aggregation operators to aggregate the different preferences where each of the element is characteristics by intuitionistic fuzzy soft numbers (IFSNs). Also, Arora and Garg (2018a) presented some prioritized weighted averaging and geometric aggregation operators by considering the dependency factor between the pairs of the attributes and the parameters during the aggregation process. Agarwal et al. (2013) introduced the concept of the generalized IFSS by incorporating the idea of generalized preference of the other senior expert. Garg and Arora (2018b) extended this concept to group-based generalized IFSS by incorporating the ideas of more than one experts' opinion into the analysis. Also, they developed some weighted averaging and geometric aggregation operators and studied their properties. Garg and Arora (2018c) developed two scaled prioritized averaging aggregation operators by considering the interaction between the membership degrees. Apart from these, under IFSS environment, some distance and similarity measures (Rajarajeswari and Dhanalakshmi, 2014; Muthukumar and Krishnan, 2016; Khalid and Abbas, 2015), entropy measures (Jiang et al., 2013), aggregation operators (Garg and Arora, 2018a, 2017a) are proposed by the researchers for solving MCDM problems under IFSS environment.

Although, the above theories are successfully applied in various situations but they have their inherent difficulties such as how to set the membership degrees in each particular case. For instance, some experts may assign the membership value as 0.2, some other assign 0.5, while rest assigns 0.7. Thus, under situations, the above mentioned techniques are unable to compute the desirable value. To manage such situations where decision makers are hesitant in expressing their preferences over alternatives, Babitha and John (2013) have introduced hesitant fuzzy soft set (HFSS) which is the hybridization of HFS and SS. For the above situation, the satisfactory membership degrees are represented by a hesitant fuzzy element $\{0.2, 0.5, 0.7\}$, which is different from a fuzzy number 0.2 (or 0.7), the interval-valued fuzzy number [0.2, 0.7] and the intuitionistic fuzzy number (0.2, 0.5). Das et al. (2017) presented the correlation coefficients for HFSS. Later on, the concept of the dual HFSS (DHFSS) (Peng and Yang, 2015; Zhang and Shu, 2016) have explored by embedding the features of the DHFS with SS. Garg and Arora (2017b) presented the distance and similarity measures for DHFSS and applied them to solve the decision making problems. The advantage of DHFSSs is to consider not only the hesitant degree of the membership but also consider simultaneously the degree of non-membership.

Presently, correlation measure is one of the most important information measure and has drawn more attention by a number of researchers under the uncertain environment. Also, DHFSSs have the great powerful ability to model the imprecise and ambiguous information in the real-world application than the other existing theories such as IFSs, HFSs, DHFSs and HFSSs. For example, if on a certain object, three experts discuss about the membership and non-membership degrees on a certain criterion and gives that preferences in terms of acceptance is 0.5, 0.3, 0.4 respectively while degree of rejection is 0.4, 0.5, 0.2. For such a case, the degrees are represented by a dual hesitant fuzzy element $(\{0.5\}, \{0.4\})$, $(\{0.3\}, \{0.5\})$ and $(\{0.4\}, \{0.2\})$ which is entirely different from the fuzzy numbers 0.5 (or 0.3), the interval-valued intuitionistic fuzzy numbers $([0.3, 0.4], [0.4, 0.5])$. Hence, the dual hesitant environment reflect all the possible opinions and appears to be a more flexible method to be valued in multifold ways according to the practical demands than the existing sets. Therefore, keeping the advantages of this set and taking the importance of the correlation measure, this paper is to derive the degree of cohesiveness among the DHFSS. As per our knowledge, the correlation measures given in aforementioned studies, however, cannot be utilized to handle the dual hesitant fuzzy soft set information. Thus, we need to propose such measures for DHFSSs. In order to achieve it, we firstly define some operational laws and their corresponding informational energies, as well as the covariance between the two DHFSSs. Then, based on these, we introduce correlation coefficient for DHFSSs and obtain some properties. Also, in order to deal with the situation where the elements in a set are correlative, weighted correlation coefficient are defined. Furthermore, we propose a decision making algorithm based on the DHFSSs and the correlation coefficients. The feasibility, as well as superiority of the proposed approach, has been illustrated with a numerical example.

To do so, the paper is summarized as follows. Section 2 presents some basic concepts related to SSs, IFSSs, HFSSs and DHFSSs. In Section 3, we introduce correlation and weighted correlation coefficients between the pairs of the DHFSSs and obtain some properties. In Section 4, we propose a decision making approach based on the correlation coefficients under the DHFSS environment where each of the element is characterized by dual hesitant fuzzy soft numbers. In Section 5, three illustrative example are discussed to validate the proposed approach and comparison analysis is shown to explore its effectiveness along with their superiority examples. Finally, Section 6 summarizes this study.

## 2. Preliminaries

This section introduces some concepts related to SSs, FSSs, IFSSs, HFSSs and DHFSSs over a universal set $U \neq \emptyset$.

**Definition 2.1** (*Zadeh, 1965*)**.** Let $U \neq \emptyset$ be a given set. A fuzzy set in $U$ is an object $A$ given by

$$A = \{\langle x, \mu_A(x)\rangle \mid x \in U\} \tag{1}$$

where $\mu_A(x)$ is a mapping from $U$ to the closed interval [0,1], and for each $x \in U$, $\mu_A(x)$ is called the degree of membership of $x$ in $U$.

**Definition 2.2** (*Xu and Yager, 2006; Xu, 2007*)**.** An intuitionistic fuzzy set in $U$ is an object $A$ given by

$$A = \left\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in U\right\} \tag{2}$$

where $\mu_A : U \to [0,1]$ and $\nu_A : X \to [0,1]$ satisfy the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$, for every $x \in X$.

**Definition 2.3** (*Torra and Narukawa, 2009; Torra, 2010*)**.** A hesitant fuzzy set $A$ in $U$ is defined as

$$A = \{\langle x, \mu_A(x^{(s)})\rangle \mid x \in U\}, \tag{3}$$

where $\mu_A : U \to [0,1]$ and $s$ represent the number of values in $\mu_A(x)$. Here, $\mu_A(x^{(s)})$ is a set of some different values in $[0,1]$, denoting the possible membership degrees of the element $x \in U$.