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Tabu search with feasible and infeasible searches for equitable coloring

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A B S T R A C T

The equitable coloring problem is a variant of the classical graph coloring problem that arises from a number of real-life applications where the cardinality of color classes must be balanced. In this paper, we present a highly effective hybrid tabu search method for the problem. Based on three complementary neighborhoods, the algorithm alternates between a feasible local search phase where the search focuses on the most relevant feasible solutions and an infeasible local search phase where a controlled exploration of infeasible solutions is allowed by relaxing the equity constraint. A novel cyclic exchange neighborhood is also proposed in order to enhance the search ability of the hybrid tabu search algorithm. Experiments on a set of 73 benchmark instances in the literature indicate that the proposed algorithm is able to find improved best solutions for 15 instances (new upper bounds) and matches the best-known solutions for 57 instances. Additional analyses show the interest of the cyclic exchange neighborhood and the hybrid scheme combining both feasible and infeasible local searches.

1. Introduction

Let $G = (V, E)$ be an undirected graph with vertex set V and edge set E. A subset I of V is an independent set if no two vertices in I are joined by an edge in E (Jin and Hao, 2015). A k-coloring of G is any partition of V into k mutually disjoint subsets (also called color classes). A k-coloring is said legal if each of its color classes is an independent set. The graph k-coloring problem is to find a legal k-coloring of G for a given k. The classical graph coloring problem (GCP) is to determine a legal k-coloring with k minimum for a general graph G (this minimum k is called the chromatic number of G and is denoted by $\chi(G)$).

In this paper, we are interested in a related coloring problem known as the *equitable coloring problem* (ECP for short) (Meyer, 1973), which adds an additional *equity constraint* to the GCP — the sizes of any two color classes differ in at most one unit. In other words, an equitable k-coloring of G (denoted by k-eqcoloring) is a legal k-coloring $\{V_1, V_2, \ldots, V_k\}$ satisfying the following equity constraint (Méndez-Díaz et al., 2015): for any two color classes V_i and V_j ($i \neq j$), $||V_i|-|V_j|| \leq 1$ where $|V_l|$ is the cardinality of color class V_l. The ECP is to determine the minimum integer k such that G admits an equitable k-coloring. This minimum k is called the equitable chromatic number of G and is denoted by $\chi_{eq}(G)$. Clearly, the chromatic number of a graph is a lower bound of the equitable chromatic number of the graph.

The ECP is known to be NP-hard in the general case (Furmańczyk and Kubale, 2005). In addition to its significance as a difficult combinatorial problem, the ECP is notable for its ability to formulate a number of practical problems. A simple example concerns the problem of assigning a set of workers to a set of tasks. Pairs of tasks may conflict each other and must not be assigned to the same worker. In addition, it is desirable that the number of tasks assigned to each worker be approximately the same for the consideration of fairness. The problem can be modeled by building a graph where a vertex corresponds to a task and an edge is created for each conflicting pair of tasks. Now if we represent the workers by colors, then this workforce assignment problem reduces to finding an equitable coloring in the corresponding graph. Other applications of the ECP arise from garbage collection (Tucker, 1973), scheduling in communication systems (Irani and Leung, 1996) and load balancing in multiprocessor machines (Furmańczyk and Kubale, 2005). An introduction to the ECP and some basic results are provided in Furmańczyk and Kubale (2004).

Despite its relevance, the ECP has been studied essentially from a theoretical point of view. For instance, Hajnal and Szemerédi (1970) proved that a graph G has an equitable $(\Delta(G)+1)$-coloring where $\Delta(G)$ is the maximum vertex degree of G. Meyer (1973) conjectured that $\chi_{eq}(G) \leq \Delta(G)$ for all connected graphs except the complete graphs and the odd cycles. This conjecture has been confirmed to be true for some special cases such as connected bipartite graphs (Lih and Wu, 1996), trees (Chen and Lih, 1994), outerplanar graphs (Kostochka, 2002; Yap and Zhang, 1997) and graphs with $\Delta(G) \geq \frac{1}{2}|V|$ or $\Delta(G) \leq 3$ (Chen et

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al., 1994). Kostochka and Nakprasit (2005) showed that graphs with an average degree up to $\Delta(G)/5$ are equitably k-colorable for every $k \geq \Delta(G)$. In addition, some special cases such as graphs with bounded treewidth (Bodlaender and Fomin, 2005) have also been identified to admit efficient polynomial algorithms or approximation algorithms.

For the purpose of practical solving of the general ECP, several exact and heuristic algorithms were recently proposed in the literature. Exact approaches have theoretical advantage of guaranteeing the optimality of the solution found, but they need a time that grows exponentially with the problem size in the general case. Still, several effective mathematical formulations and algorithms have been proposed for exactly solving the ECP. For instance, based on the asymmetric representative formulation for the GCP described in Campêlo et al. (2008), Bahiense et al. Bahiense et al. (2014) investigated the integer linear programming approach and developed effective branch-and-cut algorithms for the ECP. Méndez-Díaz et al. (2014b) adapted to the ECP the formulation and techniques used in Méndez-Díaz and Zabala (2006), studied its polyhedral structure and derived families of valid inequalities. Some of these inequalities were proven to be very effective for a cutting-plane algorithm. Very recently, Méndez-Díaz et al. (2015) proposed a DSATUR-based exact algorithm that exploits arithmetical properties inherent in equitable colorings and combines them with the techniques originally developed by Brown (1972) and Brélaz (1979) for DSATUR, and improved by San-Segundo (2012) and Sewell (1996).

On the other hand, given the intrinsic intractability of the ECP, practical heuristics have been devised to handle problem instances whose optimal solutions cannot be reached by exact approaches. Specifically, Furmańczyk and Kubale (2004) proposed two constructive greedy heuristics to generate an equitable coloring of a graph. Méndez-Díaz et al. (2014a) adapted to the ECP the implementation due to Galinier and Hao (1999) of the Tabucol algorithm initially proposed by Hertz and Werra (1987) for the GCP. Very recently, Lai et al. (2015) proposed a backtracking based iterated tabu search (BITS) algorithm for the ECP, which produced highly competitive results on a set of commonly used benchmarks.

Compared with the GCP for which a huge number of heuristic algorithms have been investigated, the ECP is surprisingly much less studied – only two recent local search approaches are available in the literature (Lai et al., 2015; Méndez-Díaz et al., 2014a). In this work, we aim to partially fill the gap by introducing a new hybrid tabu search (HTS) method that incorporates a combined use of both feasible and infeasible local searches. Our proposed hybrid tabu search is mainly motivated by two considerations. First, from the perspective of solution methods, tabu search has shown to be the most effective local search algorithm for the classical GCP (Galinier and Hertz, 2006) as well as many of its variants such as sum coloring (Jin et al., 2014) and T-coloring (Dorne and Hao, 1999). Second, from the perspective of the problem under investigation, ECP is a variant of the classical GCP where color class's equity is imposed as a strict constraint. As for strictly constrained problems, imposing solution feasibility within a neighborhood search algorithm often restricts the search process too much, while exploiting the infeasibility can help explore the search space more effectively (Aringhieri et al., in press; Chen et al., 2016b; Glover and Kochenberger, 2006; Moeini et al., 2017). Especially, methods that can tunnel through feasible and infeasible regions are particularly attractive as a means to solve these highly constrained problems (Chen et al., 2016b; Glover and Kochenberger, 2006; Qin et al., 2016). Inspired by these two observations, we proposed a hybrid TS method combining both feasible and infeasible local searches to effectively explore the search space of the ECP. The main contributions of this work can be summarized as follows.

- From the algorithm perspective, the proposed HTS approach distinguishes itself by several original features. First, we investigate a hybrid scheme which integrates both feasible and infeasible local searches within the same search algorithm in the context of solving the ECP. By relaxing the equity constraint in a controlled manner, the search algorithm is allowed to tunnel through feasible and infeasible regions to reach global optima or high quality solutions which are difficult to attain by only visiting feasible solutions. Second, to ensure an effective local optimization, we introduce a novel constrained-three-cyclic-exchange neighborhood which complements two existing neighborhoods. Third, all these ingredients are integrated in the proposed hybrid tabu search algorithm which is able to explore the solution space effectively.
- From the computational perspective, the proposed approach shows a very competitive performance on the set of 73 commonly used benchmarks. The computational results indicate that our algorithm improves the best-known solutions for 15 instances (new upper bounds) and matches the best-known results for 57 instances. Only for one instance the algorithm misses the best-known result. In particular, for several well-known DIMACS instances, our approach can significantly improve the current best-known results.

The remainder of this paper is organized as follows. In Section 2, we present the proposed algorithm for solving the ECP. Section 3 is dedicated to computational results and comparisons with state-of-the-art approaches. Section 4 investigates some important components of the HTS algorithm to understand their impacts on the performance of the algorithm. Concluding remarks are given in Section 5.

2. A hybrid tabu search algorithm for the ECP

Like the GCP (Galinier and Hertz, 2006), the ECP can be approximated by solving a series of equitable k-coloring problems (denoted by k-ECP). We start with an initial number of k colors ($k \leq |V|$) and solve the associated k-ECP. If an equitable legal k-coloring (i.e., k-eqcoloring) is found, k is decreased by one and a new k-eqcoloring is sought. This process is repeated until no k-eqcoloring can be found. The last k-eqcoloring constitutes an approximation of the minimum equitable coloring of the graph. Consequently, the ECP comes down to the problem of finding k-eqcolorings for a given k. Our HTS algorithm follows the above solution procedure and is designed to find a conflict-free k-eqcoloring from an initially conflicting k-eqcoloring. To rapidly determine an appropriate initial number of k colors that admits an equitable k-coloring, we apply the same binary search method proposed in Lai et al. (2015).

2.1. Main framework

Algorithm 1 Main scheme of the hybrid tabu search algorithm for the k-ECP

Require: Graph $G = (V, E)$, the number k of colors used
Ensure: A k-equitable coloring if found

$s_0 = greedy\_generate(G, k)$ /*Section 2.3 */
while stopping condition is not met **do**
  /*the feasible local search phase*/
  $(s_1, s_{local\_best}) \leftarrow feasible\_local\_search(s_0)$ /*Section 2.5*/
  if $f(s_{local\_best}) = 0$ **then**
    return(s_{local_best})
  end if
  /*the infeasible local search phase*/
  $(s_0, s_{local\_best}) \leftarrow infeasible\_local\_search(s_1)$ /*Section 2.6*/
  if s_{local_best} is a feasible solution and $f(s_{local\_best}) = 0$ **then**
    return(s_{local_best})
  end if
end while

Given the highly constrained feature of the k-ECP imposed by the equity constraint, one key issue to be considered is how to design search