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Development of information granules of higher type and their applications to granular models of time series

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ABSTRACT

The study is devoted to the design of information granules of higher type (especially type-2) with the use of the principle of justifiable granularity. The development of granules is realized in two key phases: first, information granules of type-1 are formed and then they are extended to type-2 constructs. Following the principle, information granules are designed by establishing a sound balance between their experimental justification (legitimacy) and specificity (associated with their underlying semantics). The definitions of coverage and specificity of type-2 information granules are revised to capture the essence of these constructs. Detailed formulas are derived for several main categories of membership functions (namely, triangular, parabolic, and square root) as well as intervals. The study delivers detailed results for interval-valued fuzzy sets described by membership functions coming from the main classes listed above. Illustrative studies include synthetic data exhibiting some probabilistic properties. The direct application of information granules of type-1 and type-2 is demonstrated in the description and prediction of time series realized in the setting of information granules (with the resulting models referred to as granular models of time series).

1. Introductory notes

Efficient ways of constructing information granules form a central item on the agenda of Granular Computing (Angelov et al., 2010; Cabrerizo et al., 2014; Pedrycz et al., 2012; Pedrycz, 2015; Pedrycz and Kwak, 2007; Sánchez and Melin, 2014; Zhou and Dai, 2015) irrespectively of the formalisms in which information granules are expressed (Park et al., 2011; Pedrycz, 2002; Pedrycz et al., 2013; Pedrycz and Izakian, 2014; Pedrycz and Vukovich, 2001; Xu and Li, 2016; Zhang et al. 2017). In the realm of fuzzy sets, we have witnessed a wealth of studies devoted to estimation techniques of membership functions (Chen and Wang, 1999; Dempe and Ruziyeva, 2012; Wang, 1994). Some of those exploit judgments resulting from human evaluations; say polling or pairwise-based techniques (such as the Analytic Hierarchy Processes, AHP) (Dopazo et al., 2014; Entani and Inuiguchi, 2015; Grobelny, 2016; Lee and Li, 2011; Singh et al., 2017; Wang et al., 2006). A commonly encountered category of methods revolves around a determination of membership functions on a basis of numeric data; here fuzzy clustering comes as a highly visible representative (Pedrycz and Rai, 2008). Recently, type-2 fuzzy sets have been an area of intensive studies being central to fuzzy modeling, classification reasoning and control (Aisbett et al., 2011; Cazarez-Castro et al., 2012; El-Nagar and El-Bardini, 2017; Hu and Wang, 2014; Lou and Dong, 2012; Mendel et al., 2010; Pedrycz and Kwak, 2006; Sun et al., 2015; Tao et al., 2012; Tavoosi et al., 2016; Wu and Tan, 2006). There is no doubt that ways of estimation of type-2 fuzzy sets is of paramount importance both conceptually as well as for various applications. Surprisingly, the studies concentrated on the systematic construction of fuzzy sets are very limited without general, systematic and convincing guidelines. Typically, type-2 fuzzy sets are established and optimized in conjunction with the design of fuzzy models, so there is no standalone estimation mechanism. This approach comes with two visible shortcomings. First, as type-2 fuzzy sets come with a larger number of parameters in comparison with those required to describe (characterize) type-1 fuzzy sets, the computing overhead becomes substantially larger thus diminishing the usefulness of the optimization mechanisms. Second, as being a part of the model, it is very likely that the semantics of fuzzy sets might not be well articulated. In conclusion, in light of these limitations, it becomes imperative to establish a sound conceptual environment of designing of information granules of higher type (Gacek and Pedrycz, 2015; Pedrycz et al., 2015).

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The principle of justifiable granularity (Pedrycz and Homenda, 2013) advocated in Granular Computing arises as a suitable alternative worth pursuing here. In a nutshell, this principle emphasizes that any information granule has to be both reflective of the experimental evidence (numeric data) and preserve its semantics by keeping an acceptable level of specificity.

It is to be noted that some initial ideas and related studies with a strong application slant were reported by Liu et al. (2017); the investigations presented here build upon the previous findings and offer further expansion. In particular, we elaborate on the refinement of the method to cope efficiently with one-dimensional data, especially their weighted versions where individual data are weighted.

The ultimate objectives of this study are succinctly outlined as follows.

First, we aim at the design of information granules of higher type, especially type-2 with the aid of the principle of justifiable granularity. The construction of information granules is realized in a stepwise hierarchical manner meaning that the type-2 information granules are built on a basis of the already constructed type-1 information granules so that the semantic continuity of the developed granules is retained.

Second, we aim at demonstrating the usage of information granules developed in this way in problems of description and prediction of time series.

The study is structured as follows. In Section 2, we briefly recall the principle of justifiable granularity, elaborate on its optimization objectives (coverage and specificity) and a nature of the underlying optimization process. Detailed computing is reported for selected categories of membership functions. In Section 3, the principle of justifiable granularity is used to construct information granules of type-2 whereas the design is carried out by expanding information granules of type-1 again formed with the aid of the same principle. In Section 4, the concept and design of granular time series are discussed. Experimental studies are reported in Section 5. Conclusions are presented in Section 6.

2. The principle of justifiable granularity — a brief overview

To make the exposure of the material self-contained, we briefly recall the main conceptual background of the principle of justifiable granularity. In essence, the principle delivers a way of building an information granule on a basis of some experimental evidence, especially numeric data, so that the resulting information granule is both experimentally justified and semantically sound. In general, by experimental justification we mean that the granule embraces (covers) the data and in this way legitimizes the existence of this granule. At the same time we anticipate that the granule is semantically meaningful and as such is kept specific enough.

The algorithm uses a set of weighted one-dimensional numeric data coming in

$$\boldsymbol{D}_w = \left\{(x_1, w_1), (x_2, w_2), \dots, (x_N, w_N)\right\} \tag{1}$$

where x_i's are the data and w_i's are the associated weights while $x_i \in \boldsymbol{R}$, $w_i \in [0,1]$, $i = 1,2,\dots, N$. If for each data, the corresponding weight is unavailable, then the dataset reads as

$$\boldsymbol{D} = \left\{x_1, x_2, \dots, x_N\right\} \tag{2}$$

where $x_i \in \boldsymbol{R}$, $i = 1,2,\dots, N$.

Denote $x_{max} = \max_{i=1,2,\dots,N}(x_i)$ and $x_{min} = \min_{i=1,2,\dots,N}(x_i)$. With the principle of justifiable granularity, the information granule A associated with the dataset $\boldsymbol{D}_w$ (or $\boldsymbol{D}$) could be formed by maximizing the performance function

$$Q = cov(A) * sp(A) \tag{3}$$

where cov (.) and sp (.) are the coverage and specificity function, respectively.

The confliction between these two criteria could guarantee the existence of the maximum point, at the same time, it ensures that the resulting granule could be simultaneously experimentally justifiable and semantically meaningful. Given the membership function of A is described as f, the parameters could be optimized by maximizing (3). To be exact, if f is an unimodal function with finite support, then it could be described by three parameters—the two end points (left bound (a) and right bound (b)) and the modal value (m). According to the discussion by Pedrycz and Wang (2016), this process is developed in the following way:

(i) the determination of the modal value. In this study, the weighted average is considered as the modal value

$$m = \frac{\sum_{i=1}^{N} x_i w_i}{\sum_{i=1}^{N} w_i}, \tag{4}$$

where $(x_i, w_i) \in \boldsymbol{D}_w$, $i = 1, 2, \dots, N$. If the dataset $\boldsymbol{D}$ is considered, then the modal value reads as

$$m = \frac{\sum_{i=1}^{N} x_i}{N}, \tag{5}$$

where $x_i \in \boldsymbol{D}$, $i = 1, 2, \dots, N$.

(ii) the optimization of bounds a and b. As these parameters could be completed in the same way (by maximizing Q), we only discuss the optimization of b here. Denote $range = (1 + r_{x_{\max}})x_{\max} - m$, $(r_{x_{max}} = \text{sign}(x_{max}))$. According to the method proposed by Pedrycz and Wang (2016), the coverage and specificity functions read as:

$$cov(A) = \sum_{k:m \le x_k \le x_{max}} \min(f(x_k), w_k) \tag{6}$$

If the weights are unknown, then

$$cov(A) = \sum_{k:m \le x_k \le x_{max}} f(x_k) \tag{7}$$

The specificity of A (with the membership function f)

$$sp(A) = \int_0^1 (1 - \frac{f^{-1}(\alpha) - m}{(1 + r_{x_{max}})x_{max} - m}) d\alpha = \int_0^1 (1 - \frac{f^{-1}(\alpha) - m}{range}) d\alpha \tag{8}$$

Then with the performance index (3), we obtain

$$b_{opt} = \text{argmax}_b Q(b) \tag{9}$$

In what follows, we present the details concerning three types of membership functions, namely linear, parabolic, and square root described by the following membership functions:

$$\text{linear} \quad L(x,a,m,b) = \begin{cases} \frac{x-a}{m-a}, & x \in [a,m] \\ \frac{b-x}{b-m}, & x \in [m,b] \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

$$\text{parabolic} \quad P(x,a,m,b) = \begin{cases} 1 - \frac{(x-m)^2}{(a-m)^2}, & x \in [a,m] \\ 1 - \frac{(x-m)^2}{(b-m)^2}, & x \in [m,b] \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

$$\text{square root} \quad S(x,a,m,b) = \begin{cases} \sqrt{\frac{x-a}{m-a}}, & x \in [a,m] \\ \sqrt{\frac{b-x}{b-m}}, & x \in [m,b] \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

We proceed with the detailed computing by involving the corresponding membership functions. (See Fig. 1.)

For the linear membership function one has

$$cov(A) = \sum_{m \le x_k \le x_{\max}} \min(\frac{b - x_k}{b - m}, w_k) \tag{13}$$

$$sp(A) = \int_0^1 (1 - \frac{b - (b-m)\alpha - m}{range}) d\alpha = 1 - \frac{b-m}{2range} \tag{14}$$