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Novel scaled prioritized intuitionistic fuzzy soft interaction averaging aggregation operators and their application to multi criteria decision making

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A B S T R A C T

Intuitionistic fuzzy soft set (IFSS) theory is one of the successful extension of the soft set theory to deal the uncertainty by introducing the parametrization factor during the analysis. Under this environment, the present paper develops two new scaled prioritized averaging aggregation operators by considering the interaction between the membership degrees. Further, some shortcomings of the existing operators have been highlighted and overcome by the proposed operators. The principal advantage of the operators is that they consider the priority relationships between the parameters as well as experts. Furthermore, some properties based on these operators are discussed in detail. Then, we utilized these operators to solve decision-making problem and validate it with a numerical example.

1. Introduction

Decision-making (DM) is one of the widely interesting topics in these days to choose the suitable alternative from a certain goal. Initially, it is assumed that the information about the alternatives is taken as a crisp number, but in the real-life situation, the collective data always contain an imprecise and vague information. In order to handle it, intuitionistic fuzzy (IF) set(IFS) (Atanassov, 1986), an extension of the fuzzy set(FS) theory (Zadeh, 1965) is one of the widely used in the field of the DM process. For instance, under this environment, Xu and Yager (2006) and Xu (2007) develops some weighted geometric and averaging operators. He et al. (2014) presented an aggregation operator with some new improved operation laws. Garg (2016c, e) presented some more generalized aggregation operators using different t-norm operations. Garg (2017h) presented an interactive aggregation operator using Einstein t-norm operations and applied them to solve the multi criteria decision making (MCDM) problems. Apart from them, some other kinds of the approaches for solving MCDM problems have been reported by the researchers in the literature (Garg, 2017d, a, 2016a, 2018; Kumar and Garg, 2016, 2017; Garg, 2017c; Rani and Garg, 2017; Singh and Benyoucef, 2011; Drissi et al., 2017; Liu et al., 2014; Ferreira et al., 2016; Zavadskas et al., 2017; Li et al., 2015; Su et al., 2017).

The above theories and their corresponding approaches are widely used by the researchers but there approaches insufficiently to consider the parameterizations tool during the decision-making process. To overcome it, Molodtsov (1999) gave the soft set (SS) theory in which preferences are noted on the different parameters. After it, Maji et al. (2001a, b) extended the SS theory under the fuzzy and IFS environment and introduced the concepts of a fuzzy soft set (FSS) and IF soft set (IFSS) respectively. Jiang et al. (2013) introduced the concepts of interval-valued IFSS by combining SS theory with the interval-valued IFS (Atanassov and Gargov, 1989). Garg et al. (2016) presented the fuzzy number IFSS and their corresponding basic operation laws. Recently, Garg and Arora (2017b) developed various distance and similarity measures for solving dual hesitant FSS MCDM problems while Garg and Arora (2017a) presented a methodology for solving MCDM problem with incomplete weight information under interval-valued IFSS environment. Garg and Arora (2018b) presented a group generalized IFSS and their aggregation operators for solving MCDM problem. By keeping the advantages of these, distance and similarity measures (Cagman and Deli, 2013; Rajarajeswari and Dhanalakshmi, 2014; Sarala and Suganya, 2016a; Muthukumar and Krishnan, 2016; Mukherjee and Sarkar, 2014; Khalid and Abbas, 2015), entropy measures (Jiang et al., 2013), aggregation operators (Arora and Garg, 2018a, b) have been proposed by the researchers for solving MCDM problems under IFSS environment.

From the above analysis, it is observed that the aggregation operators are based on the algebraic sum and product to carry out the aggregation

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process. However, all these aggregation processes do not take into account the interdependency relationship among the attributes and the parameters. Considering the importance of the relationship among the attributes, prioritized aggregation (PA) operators are one of the great significance of the decision problem. For it, Yager (2008) introduced the prioritized averaging operator. Xu and Yager (2010) and Yu (2013) investigated the multi-criteria decision-making (MCDM) based on prioritized aggregation operators. Wei (2012) investigated the PA operator under hesitant fuzzy environment. He et al. (2016a, b) proposed the scaled prioritized intuitionistic fuzzy interaction aggregation operator with consideration of the priority relationship between the different criterion. Moreover, Wang et al. (2016) developed scaled prioritized fuzzy geometric interaction averaging operator to aggregate the intuitionistic fuzzy information. Recently, Garg and Arora (2018a) presented Bonferroni mean aggregation operators under IFSS environment to solve MCDM problems.

Thus, in this paper, by keeping the advantages of PA operators and by taking the specific priority degrees of different parameters and the experts, we develop the scaled prioritized intuitionistic fuzzy soft interaction averaging (SPIFSIA) operator and its generalized form GSPIFSIA. In this proposed operators, the weighting vectors depend upon the input arguments and aggregated the values based on the priority degrees, and study their desirable properties. Then, we developed a new approach to MCDM problems under the IFSS environment based on the proposed aggregation operators. The feasibility, as well as superiority of the proposed approach, has been illustrated with a numerical example. The main research contents of this paper are divided into three parts: (i) to propose interaction operations laws, (ii) to introduce the scaled PA operator under IFSS environment and its generalized form, (iii) to establish an MCDM method based on these proposed operators. To do so, the paper is summarized as follows. In the next section, we introduce some basic concepts related to the soft set. In Section 3, we propose some new scaled averaging aggregation operators namely, SPIFSIA, GSPIFSIA based on the improved operational laws between the pairs of the intuitionistic fuzzy soft numbers. Various properties of these operators are studied in detail. In Section 4, based on these operators, we present some models for DM problems with IFSS information and demonstrated with an illustrative example. Section 6 concludes the paper with some remarks.

2. Preliminaries

The basic concepts of IFSSs are briefly reviewed over the set U and the parameter E in this section.

2.1. Intuitionistic fuzzy soft set

Definition 2.1 (*Molodtsov, 1999*). If $F : E \to P^U$, set of all subsets of U, then a pair (F, E) is called soft set(SS).

Definition 2.2 (*Maji et al., 2001a*). If $F : E \to F^U$, defined by

$$F_{e_k}(u) = \left\{ \langle u, \mu_k(u) \rangle \mid u \in U \right\}$$

where F^U be the set of all fuzzy subsets of U over the set E and $\mu_k(u)$ is the degree of membership of u over the parameter $e_k \in E$, then a pair (F, E) is called FSS.

Definition 2.3 (*Maji et al., 2001b*). If $F : E \to \mathrm{IF}^U$, is defined as

$$F_{e_k}(u_i) = \left\{ \langle u_i, \mu_k(u_i), \nu_k(u_i) \rangle \mid u_i \in U \right\}$$

where IF^U be the set of all IF subsets of U over the set E and for any $e_k \in E$, $\mu_k(u_i)$ and $\nu_k(u_i)$ are the membership and non-membership degrees respectively, with the condition that $\mu_k, \nu_k \in [0,1]$ and $\mu_k + \nu_k \leq 1$ for all $u_i \in U$, then a pair (F, E) is called IFSS. For convenience, this pair is denoted as $\alpha_{ik} = \langle \mu_{ik}, \nu_{ik} \rangle$ and called as IF soft number (IFSN).

Definition 2.4 (*Arora and Garg, 2018a*). The score function of an IFSN $\alpha_{ik} = \langle \mu_{ik}, \nu_{ik} \rangle$ is defined as

$$S(\alpha_{ik}) = \mu_{ik} - \nu_{ik} \quad ; \quad S(\alpha_{ik}) \in [-1, 1] \tag{1}$$

and an accuracy function is given as

$$H(\alpha_{ik}) = \mu_{ik} + \nu_{ik} \quad ; \quad H(\alpha_{ik}) \in [0, 1]. \tag{2}$$

Definition 2.5. Let α_{ik} and β_{ik} be two IFSNs, $S(\cdot)$ and $H(\cdot)$ be the scores values and accuracy degrees of it, respectively. By using these, the comparison law to compare IFSNs is defined as

(i) If $S(\alpha_{ik}) < S(\beta_{ik})$, then α_{ik} is smaller than β_{ik}, denoted by $\alpha_{ik} \prec \beta_{ik}$;
(ii) If $S(\alpha_{ik}) = S(\beta_{ik})$, then
  (a) If $H(\alpha_{ik}) < H(\beta_{ik})$, then $\alpha_{ik} \prec \beta_{ik}$;
  (b) If $H(\alpha_{ik}) = H(\beta_{ik})$, then α_{ik} is equivalent to β_{ik}, denoted by $\alpha_{ik} \sim \beta_{ik}$.

Definition 2.6 (*Arora and Garg, 2018a*). For a positive real number λ and three IFSNs $\alpha = \langle \mu, \nu \rangle, \alpha_{11} = \langle \mu_{11}, \nu_{11} \rangle$, $\alpha_{12} = \langle \mu_{12}, \nu_{12} \rangle$, we have

(i) $\alpha_{11} \oplus \alpha_{12} = \left\langle 1 - (1-\mu_{11})(1-\mu_{12}), \nu_{11}\nu_{12} \right\rangle$
(ii) $\alpha_{11} \otimes \alpha_{12} = \left\langle \mu_{11}\mu_{12}, 1 - (1-\nu_{11})(1-\nu_{12}) \right\rangle$
(iii) $\lambda\alpha = \left\langle 1 - (1-\mu)^\lambda, \nu^\lambda \right\rangle$
(iv) $\alpha^\lambda = \left\langle \mu^\lambda, 1 - (1-\nu)^\lambda \right\rangle$.

Based on these operations, Arora and Garg (2018a) proposed the following weighted aggregation operators for the collection of IFSNs $\alpha_{ij} (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$ with weight vectors $\eta = (\eta_1, \eta_2, \ldots, \eta_n)^T$ and $\xi = (\xi_1, \xi_2, \ldots, \xi_m)^T$ for the experts and the parameters respectively such that $\eta_i, \xi_j > 0, \sum_{i=1}^n \eta_i = 1$ and $\sum_{j=1}^m \xi_j = 1$.

(i) IF soft weighted average (IFSWA) operator

$$\mathrm{IFSWA}(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{nm}) = \left\langle 1 - \prod_{j=1}^m \left(\prod_{i=1}^n (1-\mu_{ij})^{\eta_i} \right)^{\xi_j}, \prod_{j=1}^m \left(\prod_{i=1}^n \nu_{ij}^{\eta_i} \right)^{\xi_j} \right\rangle \tag{3}$$

(ii) IF soft weighted geometric (IFSWG) operator

$$\mathrm{IFSWG}(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{nm}) = \left\langle \prod_{j=1}^m \left(\prod_{i=1}^n \mu_{ij}^{\eta_i} \right)^{\xi_j}, 1 - \prod_{j=1}^m \left(\prod_{i=1}^n (1-\nu_{ij})^{\eta_i} \right)^{\omega_j} \right\rangle. \tag{4}$$

Furthermore, Arora and Garg (2018b) utilized these operations to define prioritized aggregation (PA) operators for the collection of IFSNs α_{ij}, as follows:

(i) IF soft prioritized weighted average (IFSPWA) operator

$$\mathrm{IFSPWA}(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{nm}) = \left\langle 1 - \prod_{j=1}^m \left(\prod_{i=1}^n (1-\mu_{ij})^{\eta_i} \right)^{\xi_j}, \prod_{j=1}^m \left(\prod_{i=1}^n \nu_{ij}^{\eta_i} \right)^{\xi_j} \right\rangle \tag{5}$$

(ii) IF soft prioritized weighted geometric (IFSPWG) operator

$$\mathrm{IFSPWG}(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{nm}) = \left\langle \prod_{j=1}^m \left(\prod_{i=1}^n \mu_{ij}^{\eta_i} \right)^{\xi_j}, 1 - \prod_{j=1}^m \left(\prod_{i=1}^n (1-\nu_{ij})^{\eta_i} \right)^{\xi_j} \right\rangle \tag{6}$$