



# Conjunctive water management under multiple uncertainties: A centroid-based type-2 fuzzy-probabilistic programming approach

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## ABSTRACT

In this study, a centroid-based type-2 fuzzy-probabilistic programming (CT2FP) approach is developed for supporting conjunctive use of surface water and groundwater under multiple uncertainties. CT2FP can not only tackle uncertainties expressed as type-2 fuzzy sets (in both objective function and constraints) but also address complexity with characteristic of randomness and two-layer fuzziness (i.e., type-2 fuzzy random variables). Solution method based on  $\alpha$ -plane theory, enhanced Karnik–Mendel algorithm (EKM) and interactive algorithm are proposed to transform type-2 fuzzy-probabilistic constraints into their deterministic equivalents. A case study in Zhangweinan River Basin (China) is used to demonstrate the applicability of the proposed approach. Scenarios associated with different constraint-violation risk levels are examined to generate applicable cropping patterns and water-allocation schemes. The amount of groundwater used for irrigation can be determined (i.e., more than  $[462.84, 495.78] \times 10^6 \text{ m}^3$  in dry season and no more than  $[470.83, 537.19] \times 10^6 \text{ m}^3$  in wet season, respectively) to address the conflict between food security and ecological protection. The relationship among crop area, water allocation, and economic benefit can be reflected to enhance the agricultural sustainable development for the study basin.

## 1. Introduction

In recent years, water scarcity challenges have been exacerbated by rapid urbanization, fast industrialization, accelerated population growth and ongoing climate change. As the most water-intensive industry, agriculture accounts for 70% of the total fresh water consumption and plays an important role in world food production (Al-Ansari et al., 2015). Groundwater, a vital water resource, has been exploited to fulfil crop water demand and maximize net annual returns, especially in the regions with inadequate available surface water supplies. Excessive groundwater pumping has induced serious environmental problems, including the reduction of streamflow, the depletion of groundwater, and the degradation of ecosystem. Conjunctive use of surface water and groundwater, defined as the allocation of surface water and groundwater to achieve one or more objectives while certain constraints are satisfied, is vital to alleviate water scarcity and ensure food security (Safavi et al., 2010). Moreover, precise data is hard to be obtained due to temporal and spatial variations in agriculture system; instead, uncertainties are ubiquitous in each system component (e.g., agricultural water

demand and irrigation benefit), creating complexities in conjunctive water management. Therefore, inexact optimization techniques are desired to assist conjunctive water management under uncertainty.

Previously, a number of optimization techniques were conducted for supporting conjunctive water management under uncertainty (Sethi et al., 2006; Kerachian et al., 2010; Morankar et al., 2013; Joodavi et al., 2015; Mohammadi et al., 2016; Pastori et al., 2017; Li et al., 2017; Roy and Bhaumik, 2018). Fuzzy mathematical programming (FMP) is an attractive tool to handle epistemic uncertainty presented as type-1 fuzzy sets (Yu et al., 2017). However, the membership grades of type-1 fuzzy sets may be uncertain due to a number of economic, social, environmental, technical and political factors and it is not reasonable to use an accurate membership function for reflecting such uncertainties. For example, the values of parameters (i.e., cost for delivering water) are usually subjectively estimated by decision makers and stakeholders, and thus merely be obtained as imprecise information, such as fuzzy sets. At the same time, the estimated values from one decision maker may change under different conditions; consequently, the membership

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grades of fuzzy sets are uncertain, leading to two-layer fuzziness (type-2 fuzzy sets, T2FS).

Type reduction methods were proposed to address the above deficiencies of FMP by introducing the concept of T2FS in fuzzy logic systems (Starczewski, 2014; Cervantes and Castillo, 2015; Sanchez et al., 2015a; Castillo et al., 2016b; Ngo and Shin, 2016; El-Nagar and El-Bardini, 2017). Yeh et al. (2011) reconstructed the starting values of enhanced Karnik–Mendel algorithm to compute the centroid of interval type-2 fuzzy sets; as a result, unnecessary computations and comparisons were avoided, and type reduction could be done faster. Sanchez et al. (2015b) achieved fuzzy control through using the concept of generalized type-2 fuzzy logic system in a generalized type-2 fuzzy controller; results indicated that generalized type-2 fuzzy controllers outperform their type-1 and interval type-2 fuzzy controller counterparts in the presence of external perturbations. Golesefid et al. (2016) proposed a multi-central general type-2 fuzzy clustering approach for pattern recognitions, where the centers of clusters were considered as a set of points and the degree of belonging to the clusters was described as a general type-2 fuzzy set. Castillo et al. (2016a) presented a comparative study of type-2 fuzzy logic systems with respect to interval type-2 and type-1 fuzzy logic systems, where the theory of alpha planes and the Karnik–Mendel algorithm were used for defuzzification. Kayacan et al. (2018) tested the prediction capability of elliptic membership functions using interval type-2 fuzzy logic systems; results indicated that elliptic membership functions have comparable prediction results compared to Gaussian and triangular membership functions.

Among them, the enhanced Karnik–Mendel (EKM) algorithm can effectively deal with uncertainties presented as T2FS. However, EKM algorithm may become useless when the parameters are associated with probability distributions. Chance-constrained programming (CCP) proves to be an effective approach because that it permits violation of constraints to some extent and can handle randomness at the right-hand sides of constraints (Zeng et al., 2014). Although CCP provides means of analyzing decision risks from different constraints, it may become infeasible when the sample size is too small to obtain distributional information (Li et al., 2011). Thus, interval chance-constrained programming (ICCP) was proposed to enhance the capability of CCP by being able to deal with interval uncertainties without the requirement of known distribution functions. In real world problems, the available water for irrigation involves a number of natural process (e.g., the recharge and discharge of the aquifer) and human activities (e.g., the pumping technique to be used). It may be estimated as random variables; meanwhile, the mean value may be obtained as T2FS, leading to type-2 fuzzy random variables. Both EKM and ICCP have difficulties in tackling such hybrid uncertainties expressed as random variables with two-layer fuzziness.

Therefore, this study aims at developing a centroid-based type-2 fuzzy-probabilistic programming (CT2FP) approach for conjunctive use of surface water and groundwater in a hybrid uncertain environment. CT2FP cannot only reflect highly uncertain information (i.e., type-2 fuzzy random variables) in the right-hand sides of constraints, but also reflect relationship between system benefit and system reliability. A solution method based on  $\alpha$ -plane theory, EKM and interactive algorithm will be proposed to transform type-2 fuzzy-probabilistic constraints into their deterministic equivalents. The developed approach will be applied to a real case in Zhangweinan River Basin (China) to demonstrate its applicability. Results will be used for identifying desired decision alternatives among crop planning, agricultural irrigation, and groundwater utilization with a maximized system benefit and a minimized system-failure risk.

## 2. Methodology

In this section, the development of the CT2FP approach will be advanced. CT2FP will integrate techniques of CCP, IPP and EKM to

handle multiple uncertainties presented in terms of intervals, type-2 fuzzy sets, probabilistic distribution, and their combinations.

### 2.1. Centroid type reduction

Type-2 fuzzy sets (T2FS), as an extension of conventional fuzzy sets, are defined as sets whose membership function is also fuzzy (i.e., membership grade of each element is no longer a crisp value but a fuzzy set). A continuous T2FS  $\tilde{A}$  can be defined as (Aliev et al., 2011):

$$\tilde{A}(x) = \int_{x \in X} \int_{u \in J_x} u_{\tilde{A}}(x, u) / (x, u), J_x \subseteq [0, 1] \quad (1)$$

where  $u$  is the primary membership grade of  $x$ ,  $0 \leq u_{\tilde{A}}(x, u) \leq 1$  is the secondary membership grade of  $(x, u)$ ,  $X$  is called a primary domain and  $J_x$  is the support of the secondary membership function. In detail, a triangular type-2 fuzzy set (i.e., denoted as  $[a, b, c, d, e]$ ) can be seen as a triangular fuzzy set with triangular secondary membership function. Based on  $\alpha$ -plane theory, an  $\alpha$ -plane of a T2FS  $\tilde{A}$  can be defined as the union of the entire primary memberships of  $\tilde{A}$  whose secondary grades are greater than or equal to  $\alpha$  ( $0 \leq \alpha \leq 1$ ). Each  $\alpha$ -plane of  $\tilde{A}$  can be presented as:

$$\tilde{A}_\alpha = \{(x, u), u_{\tilde{A}}(x, u) \geq \alpha | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\}. \quad (2)$$

The  $\alpha$ -level set of the secondary membership functions at each value of  $x$  can be denoted as follows:

$$S_{\tilde{A}_\alpha}(x|\alpha) = [\inf \{u|u \in J_x, f_x(u) \geq \alpha\}, \sup \{u|u \in J_x, f_x(u) \geq \alpha\}]. \quad (3)$$

Then,  $\tilde{A}_\alpha$  can be denoted as:

$$\tilde{A}_\alpha = \int_{\forall x \in X} [\inf \{u|u \in J_x, f_x(u) \geq \alpha\}, \sup \{u|u \in J_x, f_x(u) \geq \alpha\}] / x. \quad (4)$$

Since each  $\alpha$ -plane of  $\tilde{A}$  is a special interval type-2 fuzzy set (i.e.,  $\alpha$ -level T2FS) and can be bounded by its lower and upper membership functions (i.e.,  $u_{\tilde{A}_\alpha}^-(x|\alpha)$  and  $u_{\tilde{A}_\alpha}^+(x|\alpha)$ ), the two membership functions can be formulated as:

$$u_{\tilde{A}_\alpha}^-(x|\alpha) = \int_{x \in X} \inf \{u|u \in J_x, f_x(u) \geq \alpha\} \quad (5)$$

$$u_{\tilde{A}_\alpha}^+(x|\alpha) = \int_{x \in X} \sup \{u|u \in J_x, f_x(u) \geq \alpha\}. \quad (6)$$

Then, EKM algorithm can be used to compute the centroid of each  $\alpha$ -level T2FS. Let  $x_i$  ( $i = 1, 2, \dots, N$ ) represent the discretization of  $\alpha$ -level T2FS and sorted in an ascending order (Melin et al., 2014). Centroid of  $\tilde{A}_\alpha$  (i.e.,  $C_{\tilde{A}_\alpha}^\pm = [C_{\tilde{A}_\alpha}^-, C_{\tilde{A}_\alpha}^+]$ ) can be computed as the optimal solutions of the following interval weighted average problems:

$$C_{\tilde{A}_\alpha}^- = \frac{\sum_{i=1}^L x_i \cdot u_{\tilde{A}_\alpha}^+(x_i|\alpha) + \sum_{i=L+1}^N x_i \cdot u_{\tilde{A}_\alpha}^-(x_i|\alpha)}{\sum_{i=1}^L u_{\tilde{A}_\alpha}^+(x_i|\alpha) + \sum_{i=L+1}^N u_{\tilde{A}_\alpha}^-(x_i|\alpha)} \quad (7)$$

$$C_{\tilde{A}_\alpha}^+ = \frac{\sum_{i=1}^R x_i \cdot u_{\tilde{A}_\alpha}^-(x_i|\alpha) + \sum_{i=R+1}^N x_i \cdot u_{\tilde{A}_\alpha}^+(x_i|\alpha)}{\sum_{i=1}^R u_{\tilde{A}_\alpha}^-(x_i|\alpha) + \sum_{i=R+1}^N u_{\tilde{A}_\alpha}^+(x_i|\alpha)} \quad (8)$$

where  $L$  and  $R$  are called switch points with  $x_L \leq C_{\tilde{A}_\alpha}^- \leq x_{L+1}$  and  $x_R \leq C_{\tilde{A}_\alpha}^+ \leq x_{R+1}$ . To illustrate the applicability of the centroid type reduction method, a detailed example is introduced in Fig. 1. For a triangular type-2 fuzzy set  $\tilde{A}_\alpha = [1.67, 1.90, 2.86, 3.19, 3.77]$ , the membership grades of each  $\alpha$ -level T2FS are presented. When  $N = 200$ , centroid of  $\tilde{A}_\alpha$  are  $[2.572, 2.844]$  under  $\alpha = 0$ ,  $[2.613, 2.815]$  under  $\alpha = 0.25$ ,  $[2.652, 2.786]$  under  $\alpha = 0.50$ , and  $[2.688, 2.755]$  under  $\alpha = 0.75$ , respectively.

### 2.2. Centroid-based type-2 fuzzy-probabilistic programming

In real world problems, the available water resources may be compounded by human activities (e.g., the pumping technique to be used) and expressed as fuzzy sets. These fuzzy sets are usually associated with randomness (i.e., involve a number of random processes and

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