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# Type-2 fuzzy elliptic membership functions for modeling uncertainty

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ABSTRACT

Whereas type-1 and type-2 membership functions (MFs) are the core of any fuzzy logic system, there are no performance criteria available to evaluate the goodness or correctness of the fuzzy MFs. In this paper, we make extensive analysis in terms of the capability of type-2 elliptic fuzzy MFs in modeling uncertainty. Having decoupled parameters for its support and width, elliptic MFs are unique amongst existing type-2 fuzzy MFs. In this investigation, the uncertainty distribution along the elliptic MF support is studied, and a detailed analysis is given to compare and contrast its performance with existing type-2 fuzzy MFs. Furthermore, fuzzy arithmetic operations are also investigated, and our finding is that the elliptic MF has similar features to the Gaussian and triangular MFs in addition and multiplication operations. Moreover, we have tested the prediction capability of elliptic MFs using interval type-2 fuzzy logic systems on oil price prediction problem for a data set from 2nd Jan 1985 till 25th April 2016. Throughout the simulation studies, an extreme learning machine is used to train the interval type-2 fuzzy logic system. The prediction results show that, in addition to their various advantages mentioned above, elliptic MFs have comparable prediction results when compared to Gaussian and triangular MFs. Finally, in order to test the performance of fuzzy logic controller with elliptic interval type-2 MFs, extensive real-time experiments are conducted for the 3D trajectory tracking problem of a quadrotor. We believe that the results of this study will open the doors to elliptic MFs' wider use of real-world identification and control applications as the proposed MF is easy to interpret in addition to its unique features.

## 1. Introduction

Fuzzy logic has had a significant impact on identification and control problems since it was firstly proposed by Zadeh in 1965 (Zadeh, 1965). Fuzzy logic owes its exceptional scientific reputation to its unique ability to simultaneously deal with uncertainties in the system and use the expert knowledge as an input to the fuzzy system design. As an extension of type-1 fuzzy logic systems (T1FLSs), their type-2 counterparts – type-2 fuzzy logic systems (T2FLSs) – have also made an impact on dealing with uncertainties over the last two decades. The concept of type-2 fuzzy sets first appeared in 1975 (Zadeh, 1975). Unfortunately, researchers had to wait for a while the theory to be developed more by Karnik et al. (1999). The progress of T2FLSs was primarily impeded by algorithmic and hardware limitations. Whereas the former refers to the procedure of starting from the type-2 fuzzy set and ending up with a crisp number which is called a sequence of two operations: *type reduction* and *defuzzification*, the latter refers to the relatively low computational power of processors. In particular, type reduction is challenging, because no closed form formulation exists for type-reduction as the only option is to use iterative algorithms such as Karnik–Mendel algorithm. These limitations delayed the real-time implementations of T2FLSs until the 1990s. Luckily, these limitations have diminished over the time due to several simplified algorithms which make the type reduction operation easier and relatively simpler, especially those who do not need iterative algorithms.

Since the working principle of fuzzy logic is similar to the human way of thinking, the fuzzy logic theory has been extensively implemented in function approximation and control systems. In particular, in real-time control applications, there is a significant amount of uncertainty inherent in the system due to, *inter-alia*, convoluted nonlinear dynamics in the actuators, noise from both internal and external sources, lack

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of modeling, changing working conditions as well as requirements for guaranteed stability and satisfactory performance of the system with a wide range of conditions. Intelligent, or model-free, control methodologies can address such a broad scope of challenges simultaneously, and amongst such methods, fuzzy control is one of the most successful techniques.

The theoretical basis of the interval T2FLSs, simplified version of generalized T2FLSs, was described in Mendel et al. (2006). While the third dimension in the footprint of uncertainty (FOU) of a generalized T2FLS may have an arbitrary number resulting in a non-smooth 3D geometry, the third dimension is always equal to one in an interval T2FLS. Generalized T2FLSs, in particular, the type reduction operation, are indisputably more difficult when compared to their interval counterparts. A detailed tutorial for generalized T2FLSs can be seen in Mendel (2014).

As a further simplification to T2FLSs, many type reduction algorithms have also been proposed including those does not need any type reduction phase. In turn, such simplifications and algorithmic revolutions have opened the doors of type-2 fuzzy logic controller real-time implementations within the spheres of mobile robotics (Kayacan et al., 2012), mechatronics (Kayacan et al., 2015), aerospace (Demir et al., 2016; Kayacan and Maslim, 2017; Mehndiratta et al., 2016) and any other control theory applications. The transition from T1FLS to T2FLS is still a research topic as T2FLSs appear to be a promising approach for handling uncertainties in real-world applications, which exhibit measurement noise and any other types of uncertainties (Juang and Hsu, 2009; Juang and Tsao, 2008; Sepulveda et al., 2006). The use of the T2FLSs has the potential to provide better performance when compared to a T1FLS (Sarabakha et al., 2017a). This claim does not mean that such a potential will result in a better performance every time. A high-level retrospective of T2FLSs and their type-1 counterparts can be seen in Mendel (2015).

Interval T2FLSs have tremendous potential to be used in identification, control, classification, time series prediction, signal processing, image processing and decision making systems, which have both internal and external uncertainties, by virtue of their following discriminating factors including, *inter-alia*:

- **More degrees of freedom:** Similar to the fact that T1FLSs have more degrees of freedom than the crisp systems, T2FLSs have more degrees of freedom from their design point of view.
- **Power of FOU:** There is no need to place the membership functions (MFs) precisely as there is an infinite number of type-1 MFs in the FOU of a T2FLS.
- **Handling uncertainty:** The noise rejection capability of T2FLSs is better than T1FLSs
- **Representation of uncertainty:** T2FLSs are more convenient for representing uncertainty when compared to T1FLSs

In literature, the items above are sometimes misinterpreted in a way that a T2FLS must give a more optimal response when compared to its type-1 counterpart. This is a serious paralogism because a number of papers have claimed that T2FLSs give a significant improvement when there exists a significant amount of uncertainty in the system. This conclusion can be summarized and simplified as: T2FLSs must be used when needed. A more generalized analysis can be found in Khanesar et al. (2011a).

MFs were introduced by Zadeh in his paper on fuzzy sets in 1965. The basic definitions of fuzzy logic and MFs of Zadeh were as follows:

> "*A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.*" (Zadeh, 1965).
>
> L.A. Zadeh

In the type-2 fuzzy logic research community, we characterize type-2 fuzzy MFs by the shape of the FOU. There exist a number of type-2 fuzzy MFs in literature, (*i.e.* triangular, trapezoidal, Gaussian, two-sided Gaussian, Bell-shaped, sigmoid, pi-shaped). However, all the aforementioned MFs have an adverse common feature: the parameters responsible for the support and the width are coupled. By the use of elliptic MF (Kayacan et al., 2017), the parameters responsible for the width of uncertainty are de-coupled from the parameters responsible for the center and the support of the MF. This feature allows us to analyze how the uncertainty in the input distorts the inference of the T2FLS (Khanesar et al., 2011a). Another attempt in the literature to describe a similar MF was disjunctive strictly-monotonic and conjunctive non-strictly monotonous MFs in which the user has to determine four parameters (Dombi, 1990). However, the mathematical formulation defined in this attempt is too complicated to be used in any engineering application.

Even if there are some fuzzy logic software packages which allow the user to design custom shaped MFs, there is no systematic way to choose a fuzzy MF in order to achieve a better control performance. Surprisingly, there is a relatively small literature about choosing an appropriate MFs in a T2FLS. Several membership generation methods are investigated in Medasani et al. (1998a), and the overall finding is that there is no single best method; the choice of the method depends on the particular problem. However, this work has specialized only for pattern recognition applications. Another study has investigated the effect of type-2 fuzzy MFs on modeling variation in human decision making, (Ozen and Garibaldi, 2004) and it has been shown that there is a direct relationship between the variation in decision making and the uncertainty introduced to the membership functions.

This paper aims to fill this gap by elaborating on the similarities and contrasts between the conventional type-2 fuzzy MFs and elliptic MFs. In a recent work (Muhuri and Shukla, 2017), authors have investigated a special version of elliptic MFs, which are semi-elliptic MFs. In this paper, we further extend the analysis to general elliptic MFs. In our analysis, we look at the uncertainty modeling capability of different type-2 MFs and make some comments about fuzzy arithmetic, by using the novel elliptic MF.

Since elliptic MFs have firstly been introduced, their several simulation and real-time implementations have already been published (Khanesar et al., 2015). However, the shape of the novel MF and its features have never been elaborated. The novelties of this paper are:

- Since the points, at which the MF is zero and one, are certain, the consequent part parameters can be set using the values at exactly the center of the MFs for function approximation implementations.
- From the linguistic point of view, if we are certain about what belongs to a set and what does not, we can use elliptic MFs.
- Uncertainty measures for the elliptic MF is done for the first time, and the results are compared with that of the triangular MF.
- Arithmetic operations in elliptic MFs do not change its shape.

*Paper organization.* The rest of the paper is organized as follows: In Section 2, we go over both the existing type-2 MFs and elliptic MF. In Section 3, we give a brief critique of their capability of modeling uncertainty. In Section 4, function approximation capability of the fuzzy system using elliptic type-2 MF is investigated. Uncertainty modeling capability of the type-2 fuzzy system with elliptic type-2 MF is investigated in Section 5. Function approximation capability of the type-2 fuzzy system using elliptic type-2 MF is investigated in Section 6. Section 7 presents fuzzy arithmetics by using different type-2 MFs. In Section 8, the capability of elliptic interval type-2 MFs to predict the oil price is tested. In Section 9, real-time experimental tests are conducted for the 3D trajectory tracking problem of a quadrotor unmanned aerial vehicle, in order to validate the control performances of elliptic type-2 MFs. Finally, some conclusions are drawn from this study in Section 10.