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Parameter estimation of fractional-order arbitrary dimensional hyperchaotic systems via a hybrid adaptive artificial bee colony algorithm with simulated annealing algorithm

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ABSTRACT

Hyperchaos can be observed in fractional-order nonlinear systems with suitable orders. The knowledge about systematic parameters and fractional orders is important for control and synchronization of fractional-order hyperchaotic systems. In this article, parameter estimation of fractional-order arbitrary dimensional hyperchaotic systems is investigated. Firstly, estimation of systematic parameters and fractional orders is formulated as a multi-dimensional optimization problem by treating the fractional orders as additional parameters. Secondly, a novel method called hybrid adaptive artificial bee colony algorithm with simulated annealing algorithm is proposed to deal with this optimization problem. Finally, numerical simulations and comparisons with other typical algorithms are done to demonstrate the effectiveness of the proposed algorithm, which provides a promising tool for estimation of fractional-order arbitrary dimensional hyperchaotic systems as well as other numerical optimization problems in different fields.

1. Introduction

Fractional calculus, which deals with derivatives and integrals of arbitrary order, dates back to 17th century, while its applications to physics, chemistry, engineering and other sciences have been rapidly grown only in the recent two decades (Podlubny, 1998). It has been realized that many physical systems can be elegantly described with the aid of fractional-order system theory, especially in the case of description of memory and hereditary properties of various materials and processes. During the past several years, a large number of various fractional-order systems have been proposed, such as the fractional-order Chua system (Hartley et al., 1995), the fractional-order Rössler system (Li and Chen, 2004), the fractional-order Chen system (Lu and Chen, 2006), etc.

Most of the systems we encounter in the real world are nonlinear, which can exhibit a variety of behaviors including chaos (Lorenz, 1963; Rössler, 1979a; Chen and Ueta, 1999; Lü et al., 2002) and hyperchaos (Rössler, 1979b; Wang et al., 2013; Wu and Lu, 2009). Historically, hyperchaos was firstly reported by Rössler in 1979 (Rössler, 1979b), and the most important feature of hyperchaotic system is that it has at least two Lyapunov exponents which are widely used for investigation of chaos in nonlinear autonomous systems. Compared with the traditional chaotic system, the hyperchaotic system has dynamical behavior with higher unpredictability and randomness. Therefore, the hyperchaos systems have wide applications, especially in nonlinear circuit, chaos communication, encryption, etc. In particular, during the study of fractional-order systems in recent years, it has been observed that many fractional-order differential systems also display hyperchaotic behaviors, such as the fractional-order hyperchaotic Lorenz system (Wang et al., 2013), the fractional-order hyperchaotic Chen system (Wu and Lu, 2009), the fractional-order hyperchaotic Rössler system (Li and Chen, 2004), etc.

Control and synchronization of hyperchaotic systems have been studied intensively in many engineering fields, such as chemistry, biology, signal processing and secure communication. In the past few decades, many sorts of hyperchaos synchronization have been investigated, such as complete synchronization, anti-synchronization, generalized synchronization, projective synchronization, phase synchronization, lag synchronization and so on. Particularly, there have been tremendous efforts in control and synchronization of fractional-order hyperchaotic systems due to the ubiquity of this kind of systems, which have

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wide applications in many disciplines such as in encryption, secure communication and aerospace. And many useful control schemes have been presented to synchronize two identical or different fractional-order hyperchaotic systems. For example, in Gao et al. (2015), a new fractional-order hyperchaotic system was investigated, and an active integral sliding mode control scheme was proposed to achieve the modified projective synchronization of two different fractional-order hyperchaotic systems. In Liu et al. (2014), the design of adaptive sliding mode control approach for synchronization of a class of fractional-order arbitrary dimensional hyperchaotic systems with unknown bounded disturbances was considered. In Yang and Liu (2013), the fractional-order sliding-mode control for a novel fractional-order hyperchaotic system was studied. In Liu et al. (2012), an observer-based control approach for manipulating projective synchronization of a fractional-order hyperchaotic system was put forward. In Hegazi and Matouk (2011), the analytical conditions for achieving synchronization in the fractional-order hyperchaotic Chen system via linear control were investigated by using the Laplace transform theory. In Pan et al. (2011), chaos synchronization between two different fractional-order hyperchaotic systems was achieved by utilizing feedback control method in a quite short period and both remain in chaotic state.

Most control methods mentioned above are valid only for the fractional-order hyperchaotic systems whose systematic parameters and fractional orders are known in advance. However, in the real world, the fractional-order hyperchaotic systems are usually partly known. That is, the form of the fractional-order differential equations are known, while some or all of the fractional orders and systematic parameters are unknown. Therefore, in order to control and utilize the fractional-order hyperchaos, estimating the unknown fractional orders and systematic parameters are really important. Up to now, many methods have been proposed for the parameter estimation of chaotic systems (Parlitz et al., 1997; Sun et al., 2012; Konnur, 2003; Parlitz, 1996; Li and Yin, 2014; Lin and Chen, 2014; Alfi and Modares, 2011; Lin, 2015; Wang and Xu, 2011; Wang and Li, 2010). Among the above literatures, two basic methods are mainly contained. One is synchronization-based method (Parlitz et al., 1997; Sun et al., 2012; Konnur, 2003; Parlitz, 1996), which is based on the stability analysis of chaotic systems and the corresponding control methods, but the design of both the controller and the updating law of parameters estimation is still a hard task with techniques and sensitivities depending on the considered systems. The other is the optimization method and it is a non-classical way by using evolutionary algorithms (Li and Yin, 2014; Lin and Chen, 2014; Alfi and Modares, 2011; Lin, 2015; Wang and Xu, 2011; Wang and Li, 2010). As for the second method, the unknown parameters are considered as independent variables and parameter estimation is transformed into a multi-dimensional optimization problem. Compared with the synchronization-based approach, the optimization method is not sensitive to the considered systems and easy to implement without considering any differential information, thus it is more applicable. In recent years, many evolutionary algorithms have been proposed for solving the problem, such as differential evolution algorithm (Li and Yin, 2014; Tang et al., 2012), seeker optimization algorithm (Lin and Chen, 2014), particle swarm optimization (Alfi and Modares, 2011; Yuan and Yang, 2012), artificial bee colony algorithm (Hu et al., 2015; Tien and Li, 2012), backtracking search optimization algorithm (Lin, 2015), biogeography-based optimization algorithm (Wang and Xu, 2011), quantum-inspired evolutionary algorithm (Wang and Li, 2010), etc. To the best of our knowledge, most of the works mentioned so far mainly concentrate on the parameter estimation of integer-order chaotic (hyperchaotic) systems or fractional-order traditional chaotic systems, very few has addressed the parameter estimation of fractional-order arbitrary dimensional hyperchaotic systems, which are more general and complex in the real world. Thus, in this paper, the parameter estimation problem for fractional-order arbitrary dimensional hyperchaotic systems is investigated.

In the past few decades, evolutionary algorithms (EAs) have achieved considerable success in handling complex function optimization problems as they do not depend on the differentiability, continuity, and convexity of the objective function. For instance, most of the traditional optimization methods, such as steepest decent, conjugate gradient method and Newton method, require gradient information of the objective function which make it impossible for them to deal with the non-differentiable functions. Therefore, the evolutionary algorithms have attracted more and more attentions. In the family of EAs, the most popular methods are genetic algorithms (GA) (Tam, 1992), differential evolution (DE) (Storn and Price, 1997), particle swarm optimization (PSO) (Kennedy and Eberhart, 1995; Eberhart and Shi, 2000), biogeography-based optimization (BBO) (Simon, 2008), ant colony optimization (ACO) (Socha and Dorigo, 2008), and artificial bee colony (ABC) algorithm (Karaboga, 2005). Especially, the artificial bee colony algorithm introduced by Karaboga in 2005 (Karaboga, 2005) is a biologically inspired population-based meta-heuristic evolutionary algorithm, and it imitates the foraging behavior of the real honey bees. Initially, the ABC algorithm has been used in solving unimodal and multi-modal numerical optimization problems on a limited set of test functions (Karaboga, 2005). Because it is simple in concept, easy to implement and flexible in developing hybrids and combinations with other techniques, the ABC algorithm has been successfully applied to the optimization of complex mathematical functions with or without constraints, or solving different kinds of engineering problems (Karaboga et al., 2014). In Karaboga and Akay (2009), Karaboga and Basturk (2008), the performance of ABC was compared to other novel evolutionary algorithms on well-known benchmark problems, showing that ABC was superior to other meta-heuristic algorithms such as genetic algorithm (GA), differential evolution (DE) and particle swarm optimization (PSO) on most of the instances with less control parameters. Therefore, due to the excellent properties of the ABC algorithm, in this paper, we select the ABC algorithm to solve the parameter estimation problem from the viewpoint of optimization.

On the other hand, although the evolutionary algorithms have demonstrated superior features compared to other traditional methods, there is no specific algorithm that can achieve the best solution for all optimization problems. Namely, as far as most algorithms are concerned, it is difficult to simultaneously manage the tradeoff between exploration and exploitation successfully for all the optimization problems. Similarly, the ABC algorithm is no exception. Apart from the already mentioned advantages, ABC algorithm may encounter a number of challenges (sub-optimal solutions, low robustness, slow convergence) in the optimization of composite functions, non-separable functions or problems that require constraints (Akay and Karaboga, 2012). Also, it is concluded that ABC algorithm is good at exploration but poor at exploitation in Zhu and Kwong (2010). To improve the exploitation performance of ABC algorithm, many researchers have concentrated on the study of searching strategies as they can control the balance between exploration and exploitation. Nowadays, there are two approaches mostly preferred by the researchers to improve the existing ABC algorithm, known as hybridization and modification. The former (hybridization) is the process of mixing with other EAs-based methods or traditional algorithms. For instance, in Kang et al. (2009), a hybrid simplex ABC was proposed by combining the Nelder–Mead simplex method. In Hsieh et al. (2012), a new hybrid algorithm was put forward by combining the ABC with PSO. The latter (modification) is the process of integrating an operator of an existing algorithm into the ABC. For example, in Zhu and Kwong (2010), a global-best guided ABC was introduced, which utilized the global best individual's information within the searching rule similar to PSO. In Gao et al. (2014), two new searching equations were presented to generate candidate solutions in the employed bees phase and onlooker bees phase in respect. In Imanian et al. (2014), inspired by PSO, a modified ABC algorithm is proposed by applied a new searching equation in the onlooker bees phase, which use the PSO searching strategy to guide the search for candidate solutions.