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Multi-objective immune genetic algorithm solving nonlinear interval-valued programming

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A B S T R A C T

This work studies one multi-objective immune genetic algorithm with small population to solve a general kind of unconstrained multi-objective interval-valued programming. In this optimization approach, those competitive individuals are discriminated based on interval arithmetic rules and a possibility model; a crowding degree model in interval-valued environments is developed to eliminate redundant individuals; the current population promotes different individuals to evolve towards specific directions by population sorting and immune evolution, while those elitist individuals found accelerate to explore the desired regions through genetic evolution. The theoretical analysis has showed that the computational complexity of the proposed approach depends mainly on the elitist population size. Comparative experiments have illustrated that the approach can take a rational tradeoff between effect and efficiency. It can perform well over the compared approaches as a whole, and has the potential to solving multi-modal and hard multi-objective interval-valued programming problems.

1. Introduction

In real-world multiple criteria optimization challenges, a large number of provoking problems, e.g., portfolio investment, collision detection, uncertain control systems, etc., present complex parameter uncertainty. Once their uncertain parameters are bounded, multi-objective interval programming (MIP) can be adopted to depict their attributes, usually including two general kinds of programming models, i.e., multi-objective interval number programming (MINP) and multi-objective interval-valued programming (MIVP). The former is with bounded uncertain parameters, whereas the latter involves in interval parameters. Since MIVP is a striking and provoking topic in the context of uncertain programming, any existing static multi-objective optimization algorithms become difficult in seeking its Pareto optimal solutions. The main challenge includes four points: (i) probabilistic dominance models should be designed to execute individual comparison in interval-valued environments, (ii) the Pareto optimal solutions depend greatly on the upper and lower bounds of uncertain factors, (iii) the strategies of conventional individual comparison, population diversity and population evolution cannot adapt to variable parameters environments, and (iv) although MIVP can be converted into a many-objective static programming model by means of the midpoints and radiuses of its interval-valued objectives, the computationally expensive cost is inevitable. Therefore, new intelligent optimization approaches are desired.

Despite wide applications, MIVP is a still open topic in the fields of mathematical programming and intelligent optimization. It will become increasingly popular (Jin and Branke, 2005), because lots of real-world optimization problems touch on interval parameters. Usually, the main concern involves three points: (i) since MIVP's Pareto front consists of a series of hypercubes in the objective space, new or extended individual comparison approaches and also crowding degree models are desired, (ii) some valuable biological inspirations need to be borrowed to construct effective and efficient evolutionary mechanisms, and (iii) by comparison against multi-objective single-valued mathematical programming, MIVP needs more computational resources. Although researchers made great efforts to explore efficient intelligent optimization approaches for such kind of problem, it still remains open to study useful and advanced optimization approaches capable of finding finite Pareto optimal solutions with wide coverage scope and uniform distribution. Based on this consideration,we in the present work develop a new multi-objective interval-valued immune genetic approach (MIIGA) with small population in interval-valued environments in order to solve nonlinear multi-objective interval-valued programming problems, relying upon interval arithmetic rules, simplified immune response mechanisms and additive genetic operators. The main contribution of the present work

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includes three points: (i) two interval-based computational models are designed to identify the importance and diversity of individual for a given population, which can help us solve the difficulty of individual evaluation in interval-valued environments, (ii) a competitive and hybrid multi-objective optimization algorithm is proposed to solve MIVP, which borrows some biological inspirations from immunology and genetics, and (iii) the current work is helpful for opening the door to investigate hybrid multi-objective interval-valued optimization approaches.

It is worth pointing out that, although MIIGA, like other immune genetic algorithms, is designed based on several immune metaphors from the clonal selection principle and a few inspirations presented in the genetics and evolution theory, it differs from any existing multi-objective optimization algorithms because of different problems solving and biological inspirations. Especially, whereas multi-objective immune genetic algorithms have been well studied for static multi-objective optimization problems (Luo et al., 0000), they cannot be directly applied to the case of MIVP, for which the main reason is that, on one hand, each individual should be evaluated based on interval comparison instead of real number, and on the other hand their strategies of selection, individual comparison and population diversity do not suite to multi-objective interval-valued environments. MIIGA is designed specially for MIVP problems rather than static multi-objective problems. It needs not only a computational model suitable for interval-valued environments to decide the importance of each individual in a given population, but also a new crowding degree model to measure the similarity of individual. It should be emphasized that the operators of crossover and mutation from the inspirations of genetics and evolution are only used additively to promote the process of solution search. Additionally, we also notice that immune inspirations are rarely borrowed to design optimizers for MIVP problems.

The remaining text in this paper is organized as follows: Section 2 gives a survey of related work concerning evolutionary algorithms and immune optimization for MIP. In Section 3, we describe the multi-objective interval-valued programming problem to be solved. A crowding degree model is developed in Section 4. MIIGA is formulated and designed in detail in Section 5. Further, the computational complexity of MIIGA is analyzed, while three performance criteria are cited to execute algorithm comparison in Section 6. Sections 7 and 8 present the whole experimental study, including the experimental setup, test problems, experimental analysis and an engineering application. We conclude that MIIGA is a competitive optimization tool for MIVP in Section 9.

2. Related research work

The reported studies on MIP concentrate mainly on model transformation and intelligent methodologies. Usually, several conventional approaches, e.g., interval analysis, midpoint and radius methods, weighted coefficient approaches, interval possibility models techniques and so on (Bhunia and Samanta, 2014; Cheng et al., 2014; Cheng et al., 2004; Li et al., 2011; Jiang et al., 2008; Jiang, 2005; Li et al., 2010; Yokota et al., 1995), are adopted to change uncertain objective and constraint functions into deterministic ones. Correspondingly, some achievements on model transformation have appeared in the literature. For example, Li et al. (2010) studied early how to transform MINP into a nested MIVP model by the upper and lower bounds of the objective function and constraints, and subsequently one such model was changed into a static single-objective programming model by means of the penalty function method, possibility models and midpoint and radius functions of its sub-objective interval-valued functions. This is a usual and popular model transformation method. Based on this idea of transformation, Cheng et al. (2014) discussed how MINP models were converted into unconstrained multi-objective programming ones, where the values of the uncertain objective and constraint functions for each candidate were decided by RBF neural networks, instead of Taylor series and interval analysis. Cheng et al. (2004) developed a general MINP model for process industry systems, and gave a sufficient and necessary condition which a feasible solution existed, but did not design a solution search procedure to solve one such multi-objective problem. About model transformation on MIVP, midpoint and radius functions are usually used to change MIVP as a deterministic multi-objective programming problem. For instance, Yokota et al. (1995) made an early contribution to transforming a multi-objective interval-valued integer programming problem into a static single-objective integer programming one.

The existing intelligent approaches solving MIVP models can be divided into two broader classes, i.e., model transformation approaches (Li et al., 2010; Cheng et al., 2014; Cheng et al., 2004; Yokota et al., 1995; Li et al., 2011; Bhunia and Samanta, 2014; Jiang et al., 2008; Jiang, 2005; Chen and Chen, 2014) and direct methods (Limbourg and Aponte, 2005; Zhang et al., 2008a; Sun et al., 2013; Zhang et al., 2008b; Zhang et al., 2014; Wang 2014). The former solves MIVP problems by integrating some representative multi-objective evolutionary algorithms (e.g., NSGA-II) with classical programming methods or RBF neural networks. The latter is an interval-valued intelligent optimization approach solving MIVP problems directly, for which the main challenge includes two aspects: (i) discrimination between individuals, namely identification whether an individual is superior to another one, and (ii) crowding degree models. Limbourg and Aponte (2005) based on the basic algorithmic structure of NSGA-II, suggested an extended multi-objective evolutionary algorithm (IP-MOEA) capable of being applied to MIVP problems directly. They designed an individual dominance model by using the left and right endpoints of sub-objective interval-valued functions, while a hybrid model was developed to measure the similarity of an individual to other individuals in a given population. Additionally, studies on multi-objective evolutionary algorithms, based on possibility degree models have achieved initial success. Gong et al. (2010) and Zhang et al. (2008b) proposed two valuable multi-objective optimization algorithms related to either NSGA-II or particle swarm optimization (PSO), in which some interesting models of probabilistic dominance and crowding degree were designed to identify those valuable individuals and compute their similarity degrees. In order to find the most preferred solutions for MIVP problems, Sun et al. (2013) designed a metric index to emphasize the approximation relationship between candidate solutions and Pareto optimal solutions, and then developed an interactive evolutionary algorithm after incorporating an optimization-cum-decision-making procedure and the idea of preference selection. Zhang et al. (2008a) studied a multi-objective particle swarm optimization algorithm to handle MIVP problems without interval constraints. In one such algorithm, a probable dominance method, used in processing intervals was proposed to identify the qualities of solutions.

3. Problem formulation and basic concepts

Let IR represent the set of bounded and closed intervals in R. Each element in IR is said to be an interval number. Consider the following general nonlinear multi-objective interval-valued programming problem of form MIVP:

$$\min_{\mathbf{x}\in D} f(\mathbf{x},U) = (f_1(\mathbf{x},U), f_2(\mathbf{x},U), \ldots, f_m(\mathbf{x},U)),$$

with bounded and closed domain D in R^p, and decision vector $\mathbf{x} \in D$, where U denotes a q-dimensional interval vector, i.e., the product of q interval numbers; $f(\mathbf{x},U)$ is the interval-valued vector function. $f_i(\mathbf{x},U)$ is the ith sub-objective function with $1 \le i \le m$, represented by $[\underline{f_i}(\mathbf{x}), \bar{f}_i(\mathbf{x})]$ by means of interval arithmetic rules.

Interval analysis is a valuable tool in the fields of interval programming and dynamic systems. Moore et al. (2009) introduced some basic properties on interval number. They gave the following partial order relation between two interval numbers $A = [A^L, A^R]$ and $B = [B^L, B^R]$,

$$A < B \Longleftrightarrow A^R < B^L. \quad (1)$$