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A review on stability analysis of continuous-time fuzzy-model-based control systems: From membership-function-independent to membership-function-dependent analysis

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A R T I C L E I N F O

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A B S T R A C T

This paper reviews the stability analysis of continuous-time fuzzy-model-based (FMB) control systems, with emphasis on state-feedback control techniques, which is an essential issue received a great deal of attention in the fuzzy control community. It gives an overview of the milestones and the trend of developments and achievements for the past decades. Focusing on the stability analysis of FMB control systems, it summarizes the issues in the four fundamental and essential aspects, namely, the types of membership-function matching, types of Lyapunov functions, types of stability analysis and the techniques of stability analysis are discussed. To start with systematic discussion, the FMB control systems are categorized into three types of membership-function matching, namely, perfectly, partially and imperfectly matched premises, regarding the premise membership functions and the number of rules used in the fuzzy model and fuzzy controller which forms the FMB control system. The features of each category are thoroughly discussed from theoretical to practical point of view. Various types of Lyapunov functions available in the literature for conducting stability analysis and their characteristics are then reviewed. Especially, the focus of this paper is to promote the concept of membership-function-dependent (MFD) stability analysis, which makes use of the information of membership functions aiming to relax the stability conditions compared with the dominant membership-function-independent (MFI) stability analysis in the literature. The techniques for MFI and MFD stability analysis are then discussed in details, which provide some solid ideas to analyze the stability of FMB control systems. More importantly, it sheds light on the fact that MFD stability analysis demonstrates a greater potential than the MFI one for relaxing the conservativeness of stability analysis results, which points a promising research direction for this topic. The purposes of this paper are to provide a comprehensive update for the stability analysis of FMB control systems to the researchers in the field and serve as a quick guide for the potential researchers who want to enter the field.

1. Introduction

Since the introduction of fuzzy sets in 1965 by Prof. Lotfi A. Zadeh (Zadeh, 1965), it has been rapidly developed to a promising research. By using linguistic rules, human spirit and knowledge can be incorporated into a fuzzy logic system. With the support of fuzzy set theory and mathematics, a fuzzy logic system can perform reasoning according to the designated linguistic rules. Fuzzy logic systems were used successfully in a wide range of areas and applications (Sugeno, 1985; Precup and Hellendoorn, 2011; Yager and Zadeh, 2012) such as assessment (Naji and Ramdani, 2016), classification (Nguyen and Nahavandi, 2016; Ekong et al., 2016; Korytkowski et al., 2016), control (Tanaka and Wang, 2001; Lam and Leung, 2011; Lam, 2016), decision making (Kilincci and Onal, 2011; Chan et al., 2015; de Salles et al., 2016; Bilgin et al., 2016; Korczak et al., 2016; Darwish and Abdelghany, 2016), evaluation (Ozdemir and Tekin, 2016; Chai et al., 2016), forecasting (Torrini et al., 2016; Mekanik et al., 2016; Ye et al., 2016), learning (Chi and Liu, 2016; Wang et al., 2016b; Almaraashi et al., 2016), modeling (Abbasov and Shahbazova, 2016; Sheehan and Gough, 2016) and etc.

Fuzzy control is a hot research topic, which has drawn a great deal of attention from researchers working from fundamental research to domestic/industrial applications in the past few decades. It has been witnessed that fuzzy control has developed from model-free approach (Mamdani, 1993) to model-based approach (Sala et al., 2005;

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Feng, 2006; Guerra et al., 2015), from the author's view with focus on continuous-time systems and state-feedback control technique, which can be summarized by four stages as shown in the upper half section of Fig. 1 and some milestones happened during the development are marked in the lower half section of Fig. 1.

Stage 1 happened during early 1970s to late 1980s, where the concept of fuzzy control has been developed. Mamdani-type fuzzy logic controller (Mamdani and Assilian, 1975; Mamdani, 1976, 1977) is an example, which consists of four basic units namely fuzzifier, rule base, inference engine and defuzzifier. It collects expert knowledge from the control problem and incorporates them using fuzzy sets and linguistic rules. Thanks to the fuzzy logic, the fuzzy logic controller can mimic the human spirit to reproduce the human control actions. As a result, it demonstrates that the fuzzy logic controller can handle ill-defined and complex nonlinear control problems well without the need of any mathematical models. So it is termed as a model-free control approach. Since then, a lot of successful applications have been reported such as sludge wastewater treatment (Tong et al., 1980) and control of cement kiln (Holmblad and Ostergaard, 1981) found in 1980s, and then regulation of DC–DC power converters (So et al., 1996; Leung et al., 1998), motor control (Guillemin, 1996) in 1990s. However, the model-free approach suffers from (1) the design is heuristic which is a time-consuming design process and may lead to inconsistent performance, (2) the system stability and robustness are not guaranteed but tested experimentally.

Stage 2 started mainly from early 1990s to mid-2000s, where the fuzzy-model-based (FMB) control concept kicked in. The Takagi–Sugeno (T–S) or also known as Takagi–Sugeno–Kang (T–S–K) fuzzy model (Takagi and Sugeno, 1985; Sugeno and Kang, 1988) plays an important role to support the system analysis and control design. It describes the dynamics of nonlinear system as an average weighted sum of some local linear sub-systems where the weights characterized by membership functions measure the contribution made by each. In general, there are three main approaches for constructing a T-S fuzzy model: (1) Applying system identification techniques to experimental data (Takagi and Sugeno, 1985; Sugeno and Kang, 1988), (2) Applying sector nonlinear techniques (Kim et al., 1997) to mathematical model, (3) Approximating the nonlinear system by combining linearized models at the chosen operating points with membership functions (Tsai, 2013).

By connecting a state-feedback fuzzy controller (Tanaka and Sugeno, 1992; Chen et al., 1993; Tanaka et al., 1996; Wang et al., 1996) (without otherwise stated, referred to as fuzzy controller hereafter) to a nonlinear plant represented by the T-S fuzzy model in a closed loop, an FMB based control system is formed. As the fuzzy controller is represented as an average weighted sum of linear state-feedback sub-controllers, the FMB control system is expressed as an average weighted sum of linear control sub-systems formed by the local linear sub-systems from the T-S fuzzy model and the linear state-feedback sub-controllers from the fuzzy controller.

In stage 2, the stability of FMB control systems were studied through mainly investigating the linear control sub-systems. Basic stability conditions in terms of linear matrix inequalities (LMIs) (Boyd, 1994; El Ghaoui and Niculescu, 2000) were obtained, where a feasible solution (if any) can be found numerically using convex programming techniques (Tanaka and Sugeno, 1992; Chen et al., 1993). Sector nonlinearity technique was then proposed to provide a systematic approach to construct a T-S fuzzy model for the nonlinear system based on its mathematical model (Tanaka et al., 1996; Wang et al., 1996). An important parallel distributed compensation (PDC) concept (Tanaka et al., 1996; Wang et al., 1996) was proposed to relax the stability analysis results. The PDC design concept suggests that the fuzzy controller shares the same premise membership functions and the same number of rules from the T-S fuzzy model, which facilitates the stability analysis by grouping the same cross terms of membership functions possessed by the linear control sub-systems. Along the same line of analysis, further relaxed results were reported in Kim and Lee (2000), Liu and Zhang (2003b, a), Teixeira et al. (2003) and Fang et al. (2006) by using different grouping ways of cross terms and with the introduction of slack matrices and then generalized in Sala and Ariño (2007a) by considering the permutations of membership functions using Pólya theorem. Further follow-up work based on Pólya theorem can be found in Montagner et al. (2009) and Lo and Wan (2010). The aforementioned PDC results were obtained based on a common quadratic Lyapunov function, a non-PDC design concept was proposed in Guerra and Vermeiren (2004) where the Lyapunov function depending on the membership functions was used.

Some more stability analysis work based on T-S fuzzy model has continued after the period of stage 2 but it is less active comparatively. It is worth mentioning that the introduction of sum-of-squares (SOS) concept (Prajna et al., 2004) inspires the development of polynomial fuzzy-model-based (PFMB) control systems (Tanaka et al., 2009b; Sala and Ariño, 2009; Sala, 2009). A polynomial fuzzy model was proposed in Tanaka et al. (2009b), which extends the T-S fuzzy model to represent a wider class of nonlinear plants by allowing polynomials in the local sub-systems. Along the line of PDC design concept, stability conditions in terms of SOS were obtained in Tanaka et al. (2009b). Since then, a lot of research on stability analysis of PFMB control systems have been carried on and variations of SOS-based stability conditions have been obtained for different control problems, just to name a few, such as observer-based control problems (Tanaka et al., 2012; Han et al., 2014b; Liu and Lam, 2015; Liu et al., 2016; Chibani et al., 2016), output-feedback control problems (Lam, 2011a), positive control problems (Li et al., 2016), regulation control problems (Lam and Lo, 2013), sampled-data control problems (Lam, 2012), stabilization control problems (Tanaka et al., 2009b, a; Sala and Ariño, 2009; Lam and Narimani, 2009b; Narimani and Lam, 2010; Lam and Seneviratne, 2011; Lam, 2011b; Narimani et al., 2011; Lam et al., 2012, 2013b; Guelton et al., 2013; Cao et al., 2014a; Lam and Tsai, 2014; Liu et al., 2014a; Chen et al., 2014; Lam et al., 2015a; Chen et al., 2015b; Lam, 2016; Tanaka et al., 2016; Yu et al., 2016), switching control problems (Lam et al., 2013b), tracking control problems (Lam and Li, 2013) and etc.

Generally speaking, the T-S FMB control system separates the closed-loop control system into the linear (linear control sub-systems) and nonlinear (membership functions) parts. Consequently, the stability analysis, which can be made easy by mainly investigating the linear part through the permutations of membership functions, are considered under the PDC design concept. However, the information of membership functions is not considered in the stability analysis and neither in the stability conditions. It is thus the author terms this stability analysis to be membership-function-independent (MFI). As the stability conditions are MFI, it means that the MFI stability analysis is for a family of FMB control systems with the same set of linear control sub-systems but any membership functions, which explains the source of conservativeness. The PDC is a dominant concept in the MFI stability analysis due to the permutations of the cross terms of membership functions can facilitate the analysis. However, the PDC design concept requires that the fuzzy model and fuzzy controller share the same sets of premise membership functions and the same number of rules, which impose limitations on various issues such as design flexibility, computational demand, implementation complexity, robustness property, analysis feasibility and applicability. The author advocates the concept of perfectly, partially and imperfectly matched premises (Lam and Leung, 2011), which categories the FMB control systems into three categories according to the premise membership functions and number of rules used in the fuzzy model and fuzzy controller. The three categories cannot be superseded by one or another due to each category demonstrates its own limitations and benefits. Membership-function-dependent (MFD) stability analysis makes possible and unifies the stability analysis for these three categories of FMB control systems. Also, utilizing the information of membership functions in the stability analysis provide an efficient way to offer more relaxed stability analysis results.

Stages 3 and 4 move from MFI stability analysis to MFD stability analysis where stage 3, happening from mid-2000s to late 2000s, focuses

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