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Evolutionary identification of hidden chaotic attractors

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ABSTRACT

In this participation we discuss the possibility of mutual fusion of evolutionary algorithms and deterministic chaos. As demonstrated in previous research papers, evolutionary algorithms are capable of chaotic system control, identification or synthesis and vice versa, chaos can be observed in the evolutionary dynamics. More exactly, in this paper there is numerically demonstrated possible solution of the question whether identification of so-called basin of attraction for hidden attractor can be done by evolutionary algorithms. Hidden attractors are a special kind of attractors, that are hidden in the system structure and if ignored (undiscovered), then can cause serious damages, as already observed in the real world. The research presented here is bivalent. At first it shows, that evolutionary algorithms are able to identify presence of hidden attractors in the system, but also it can be extended to study an existence of hidden attractors in the evolutionary algorithms dynamics. All numerical simulations are demonstrated on Chua's chaotic attractor that contains an example of hidden attractor and at the end there are discussed discrete systems (synthesized by evolution) that likely exhibit hidden attractors, too.

1. Introduction

Evolutionary algorithms (EAs) and deterministic chaos which is complex behavior produced by complex as well as simple dynamical systems, are tightly joined and create interdisciplinary fusion of two interesting areas. This paper discusses use of EAs on numerical identification of the so-called hidden attractors, that are part of chaotic dynamics. To understand this very specific topic, it is important to explain a little the importance of hidden attractors for industrial technology and also for dynamics of evolutionary algorithms, as well as previous use of EAs on chaotic dynamics control, identification and synthesis. In this section we will overview consequently (a) hidden attractors, (b) EAs use on chaotic dynamics and (c) existence of chaos inside EAs and its impact on EAs performance.

The hidden attractors (see Fig. 4) are a special set of points that reflect dynamic of observed system as reported in Kuznetsov et al. (2010), Leonov et al. (2011), Leonov et al. (2012), Leonov and Kuznetsov (2013), Kuznetsov et al. (2013), Leonov et al. (2012), Bragin et al. (2011), Leonov et al. (2011), Chen et al. (2015), Bao et al. (2015), Chen et al. (2015), Li et al. (2014), Kuznetsov and Leonov (2014), and Kapitaniak and Leonov (2015). In general and from a computational point of view attractors can be regarded as self-excited and hidden attractors. Self-excited attractors can be localized numerically by a standard computational procedure, in which after a transient process a trajectory, starting from a point of unstable manifold in a neighborhood of an equilibrium, reaches a state of oscillation, therefore one can easily identify it. In contrast, for a hidden attractor, a basin of attraction does not intersect with any small neighborhoods of equilibria. Hidden attractor can be chaotic as well as periodic solution – e.g. the case of coexistence of the only stationary point which is stable and a stable limit cycle (like in the counterexamples to the Kalman and Aizerman conjecture) (Kuznetsov et al., 2010; Leonov et al., 2011, 2012; Leonov and Kuznetsov, 2013; Kuznetsov et al., 2013; Leonov et al., 2012; Bragin et al., 2011; Leonov et al., 2011; Chen et al., 2015; Bao et al., 2015; Chen et al., 2015; Li et al., 2014; Kuznetsov and Leonov, 2014; Kapitaniak and Leonov, 2015). Classical attractors are self-excited, attractors can therefore be obtained and identified numerically by the standard computational procedure as for example for the Lorenz system. It can easily predict the existence of self-excited attractor, while for hidden attractor the main problem is how to predict its existence in the phase space. Thus, for localization of hidden attractors it is important to develop special procedures, since there are no similar transient processes leading

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to such attractors. If the hidden attractor is present in the system dynamics and if coincidentally reached, then device (airplane, electronic circuit, etc.) starts to show quasi-cyclic behavior, that can, based on kind of device, cause real disasters. As an example can be used Gripen jet fighter crash[1] or F-22 raptor crash landing[2] caused by computer malfunction that lead into oscillations (called also wind-up in control theory). Hidden attractors as a part of deterministic chaos, can be studied in Kuznetsov et al. (2010), Leonov et al. (2011), Leonov et al. (2012), Leonov and Kuznetsov (2013), Kuznetsov et al. (2013), Leonov et al. (2012), Bragin et al. (2011), Leonov et al. (2011), Chen et al. (2015), Bao et al. (2015), Chen et al. (2015), Li et al. (2014), Kuznetsov and Leonov (2014), Kapitaniak and Leonov (2015) and deterministic chaos itself for example in Hilborn (1994), Kantz and Schreiber (1997), Wright and Agapie (2001). The latest research papers, that discuss hidden attractor topics and which are source of very good information are Leonov et al. (2014), Leonov et al. (2015), Leonov et al. (2012), Leonov and Kuznetsov (2013), Sharma et al. (2015), Jafari and Sprott (2013), Jafari et al. (2015), Molaie et al. (2013), Pham et al. (2014), Pham et al. (2014), Sprott et al. (2015), Tahir et al. (2015). In those papers there are discussed topics like controlling of hidden attractors (Sharma et al., 2015) and/or its theoretical background as for example in Leonov and Kuznetsov (2013), Molaie et al. (2013) or Sprott et al. (2015).

In the last 15 years it has been demonstrated that evolutionary algorithms can be used successfully in deterministic chaos system control, its identification and/or synthesis. Deterministic chaos control, see Schuster (1999) (this handbook can serve as a very good reference book to that topic and related areas) and control law synthesis is another area of EAs use. The interest in the control of chaotic systems has been an active area of research during the past decade. Numerous papers focused on chaos control with EAs are published frequently. As an example we can mention paper (Ott et al., 1990) where the basic ideas about chaos control, or about CML systems control, that is more complex and usually expect some preliminary information to derive control law (for classical controllers) were published. One of the first and important initial studies, of EAs for control by means of EAs (including CML systems control) use was reported in Zelinka et al. (2007), Zelinka et al. (2006) and Zelinka et al. (2006), where the control law was based on the Pyragas method: Extended delay feedback control – ETDAS Pyragas (1995). Those papers focused on the tuning of several parameters inside the control technique for a chaotic system. Some research in this field has been recently done using the evolutionary algorithms for optimization of local control of chaos based on a Lyapunov approach (Richter and Reinschke, 2000; Richter, 2002). But the approach by EA described in this paper is very different from the classical one due to EA principles. EA searched for optimal setting of adjustable parameters of arbitrary control method to reach desired state or behavior of chaotic system. Another applications of unconventional control of chaotic systems by EA are described in Zelinka et al. (2011) and Oplatkova et al. (2010), Oplatkova et al. (2010), Oplatkova et al. (2010). EA synthesis of control law for discrete chaotic system is discussed there. Compared to that, published paper (Oplatkova et al., 2010, 2010, 2010) shows possibility how to generate the whole control law (not only to optimize several parameters) in order to stabilize a chaotic system. The synthesis of control is inspired by the Pyragas's delayed feedback control technique (Just et al., 1999; Pyragas, 1992). Unlike the original OGY control method (Ott et al., 1990), it can be simply considered as a targeting and stabilizing algorithm together in one package. Another big advantage of the Pyragas method is the amount of accessible control parameters. Methods used in generating new chaotic systems from physical systems or from“manipulations” (e.g., control and parameter estimation) (Grebogi and Lai, 1999; Hu et al., 1999) are based on classical mathematical analysis. Along with these classical methods, EAs are also applicable on chaos system synthesis, as reported in Zelinka et al. (2008), Zelinka et al. (2010). Both papers introduce the chaos synthesis by means of novel EA method. This method is similar to genetic programming or/and grammatical evolution. As results in Zelinka et al. (2008), Zelinka et al. (2010) show that such approach is able to synthesize new and "simple" chaotic systems based on some elements contained in a pre-chosen existing chaotic system and a properly defined cost function. The research in Zelinka et al. (2010) consists of 11 case studies: the aforementioned three EAs in 11 versions. For all the algorithms, 100 simulations of chaos synthesis were repeated and then averaged to guarantee the reliability and robustness of the proposed method. The most significant results are carefully selected, visualized and reported in Zelinka et al. (2010).

Also few research papers have demonstrated that chaos can also be observed in evolutionary dynamics (Wright and Agapie, 2001; Zelinka et al., 2010), or used instead of pseudorandom number generators like the logistic map like Persohn and Povinelli (2012) who is not the only one who used the logistic map. Another paper (Drutarovsky and Galajda, 2007) discusses use of logistic map like chaos-based true random number generator embedded in reconfigurable switched-capacitor hardware. Xing in Wang and Xue (2012) proposed an algorithm of generating pseudorandom number generator and combined the couple map lattice (Schuster, 1999) and chaotic iteration. Authors also tested this algorithm in NIST 800-22 statistical test suits and it was used in image encryption. In Pareek et al. (2010) authors investigate interesting properties of chaotic systems in order to design a random bit generator (called CCCBG) in which two chaotic systems are cross-coupled with each other. For evaluation of the bit streams generated by the CCCBG, the four basic tests are performed: monobit test, serial test, auto-correlation, and Poker test. Also the most stringent tests of randomness: the NIST suite tests have been used. Several studies have already dealt with the possibilities of integration of chaotic systems into the PSO algorithm and the performance of such algorithms, see Araujo and Coelho (2008), Alatas et al. (2009). Papers (Pluhacek et al., 2013, 2012, 2012) extends the previous experiments of Alatas et al. (2009) and investigates the impact of using different chaotic maps on the behavior of PSO algorithm especially in terms of convergence speed and premature convergence risk. Three different chaotic systems (maps) are used and their impact is compared in this study. The aim is to find a link between specific chaotic system and specific behavior of the PSO algorithm.

Above mentioned information and references are only a fraction of existing research papers that discuss mutual fusion of EAs and chaos dynamics. In this paper there is discussed possibility on EA identification of hidden attractor existence, because, as written in many papers and books (e.g. Schuster, 1999; Hilborn, 1994), basin of attraction (i.e. set of "start" points that lead system dynamics into hidden attractor) is very hardly identifiable. This is also discussed in papers (Kuznetsov et al. (2013)) and Bragin et al. (2011) that present identification of basin of attraction for hidden attractor (so in fact the hidden attractor existence) by means of classical numerical algorithms. Identification by EAs is based on suitable cost function definition that expresses quality of system state trajectory and that can be visualized as a surface. The global extreme on surface then represents optimal solution (or set of start/initial points "leading" trajectories to the hidden attractor). As an example of complexity of such surfaces can be used in Fig. 1, see also Zelinka et al. (2010). From the geometry of such surface is

[1] https://www.youtube.com/watch?v=jP-QMmzGL5I
[2] https://www.youtube.com/watch?v=faB5bIdksi8