A selection method for evolutionary algorithms based on the Golden Section

Erik Cuevas [a,b,*], Luis Enríquez [a], Daniel Zaldívar [a,b], Marco Pérez-Cisneros [a]

[a] *Departamento de Electrónica, Universidad de Guadalajara, CUCEI, Av. Revolución 1500, Guadalajara, Mexico*
[b] *Centro Tapatío Educativo A.C, Av. Juárez 340, Colonia Centro, Guadalajara, Mexico*

A R T I C L E   I N F O

A B S T R A C T

During millions of years, nature has developed patterns and processes with interesting characteristics. They have been used as inspiration for a significant number of innovative models that can be extended to solve complex engineering and mathematical problems. One of the most famous patterns present in nature is the Golden Section (GS). It defines an especial proportion that allows the adequate formation, selection, partition, and replication in several natural phenomena. On the other hand, Evolutionary algorithms (EAs) are stochastic optimization methods based on the model of natural evolution. One important process in these schemes is the operation of selection which exerts a strong influence on the performance of their search strategy. Different selection methods have been reported in the literature. However, all of them present an unsatisfactory performance as a consequence of the deficient relations between elitism and diversity of their selection procedures. In this paper, a new selection method for evolutionary computation algorithms is introduced. In the proposed approach, the population is segmented into several groups. Each group involves a certain number of individuals and a probability to be selected, which are determined according to the GS proportion. Therefore, the individuals are divided into categories where each group contains individual with similar quality regarding their fitness values. Since the possibility to choose an element inside the group is the same, the probability of selecting an individual depends exclusively on the group from which it belongs. Under these conditions, the proposed approach defines a better balance between elitism and diversity of the selection strategy. Numerical simulations show that the proposed method achieves the best performance over other selection algorithms, in terms of its solution quality and convergence speed.

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1. Introduction

Expert systems (Jackson, 1998) are approaches commonly adopted to support decision-making processes and problem-solving applications. Some of their main characteristics include their ability to solve complex problems and their capacity to produce consistent decisions. One of the most critical operations, in the decision-making process, is the evaluation or ranking of all possible alternatives of action in order to find the best solution for a particular problem. Therefore, optimization algorithms work cooperatively with expert systems schemes in the efficient search of potential solutions in the decision-making process.

Evolutionary algorithms (EAs) (De Jong, 2006) are optimization methods based on the model of natural evolution. In general, Genetic Algorithms (GA) (Goldberg, 1989) are the most popular representatives of such techniques. EAs operate on a population $\mathbf{P}^k(\{\mathbf{p}_1^k, \mathbf{p}_2^k, \ldots, \mathbf{p}_N^k\})$ of N candidate solutions (individuals) that evolve from the initial point ($k=0$) to a total *gen* number of iterations ($k=gen$). A candidate solution $\mathbf{p}_i^k$ ($i \in [1, \ldots, N]$) represents a d-dimensional vector $\{p_{i,1}^k, p_{i,2}^k, \ldots, p_{i,d}^k\}$ where each dimension corresponds to a decision variable of the optimization problem. In each iteration, a set of stochastic operations are applied over the population $\mathbf{P}^k$ to build the new population $\mathbf{P}^{k+1}$. Such operations are mutation, recombination, and selection. The quality of each candidate solution $\mathbf{p}_i^k$ is evaluated by using an objective function $f(\mathbf{p}_i^k)$ whose final result represents the fitness value of $\mathbf{p}_i^k$.

The effect of the evolutionary operators in the search strategy has been extensively demonstrated and documented (Hancock et al., 1994). Mutation incorporates modifications in the population as a mechanism to escape of local optima. Recombi-

* Corresponding author at: Departamento de Electrónica, Universidad de Guadalajara, CUCEI, *Av.* Revolución 1500, Guadalajara, Mexico.
E-mail addresses: erik.cuevas@cucei.udg.mx, cuevas@inf.fu-berlin.de (E. Cuevas), daniel.zaldivar@cucei.udg.mx (D. Zaldívar), marco.perez@cucei.udg.mx (M. Pérez-Cisneros).

Table 1
Comparative overview of classical selection methods.

Method	Characteristic	Disadvantage
Proportional method (Holland, 1975)	It assigns a selecting probability to each individual in terms of its relative fitness value.	Problems in case of negative objective functions values or minimization tasks.
Tournament selection (Blickle et al., 1995)	It selects the best solution for a set of q different individuals.	Low selective pressure that promotes extensively the exploration, but adversely punishes the exploitation.
Linear ranking (Darrell, 1989)	It uses a linear function to map the ranking index of each solution to a selection probability.	Solutions with the same fitness value can obtain a very different selection probability affecting the search strategy.

nation interchanges essential characteristics of the search space among individuals. Selection (Bäck, 1992) conducts the search strategy towards promising regions of the search space by the use of information currently available in the population. Both Mutation and recombination (Bäck, 1992) allow the exploration of the search space while selection exploits the information already present in the population with the objective of improving it.

The balance between exploration and exploitation (Nandar & Ponnuthurai, 2015) in a search strategy can be interpreted, as the conflicting action of increasing the solution diversity and simultaneously refining the solutions, already known, which mainly maintain the best fitness values. In EAs, this balance is critical in order to achieve a good performance of the search strategy. Under such circumstances, the selection operator provides an important mechanism to modify the relation exploration-exploitation (Bäck & Hoffmeister, 1991). To increase the intensity of selecting individuals with high fitness values augments the exploitation of the search strategy (Bäck, 1994). On the other hand, decreasing the emphasis on selecting such individuals permits the selection of low-quality solutions. In these conditions, the exploration of the optimization strategy is promoted (Baker, 1987).

Different selection techniques have been proposed in the literature with different performance levels. The most popular methods are the Roulette method (Holland, 1975), the tournament selection (Blickle, Thiele, & Eshelman, 1995) and the linear ranking (Darrell, 1989). The proportional method assigns a selecting probability to each individual regarding its relative fitness value. This mechanism presents several flaws in case of negative objective functions or minimization tasks (Noraini &, Geraghty, 2011). In the tournament selection technique, it is selected the best solution of a set of q different individuals randomly obtained from the whole population. The standard size of the tournament set is $q = 2$. Under tournament, it has been demonstrated that the selective pressure is very low, favoring the exploration extensively and punishing the exploitation of the search strategy (Brad & Goldberg, 1995). Finally, the linear ranking method uses a linear function to map the ranking index of each solution to a selection probability. Although linear ranking maintains a good selective pressure, the method presents a critical difficulty. Since linear ranking assigns a selection probability to each solution depending on its respective ranking index, two solutions with the same fitness value can obtain a very different selection probability. This inconsistency adversely affects the performance of the search strategy during the optimization (Blickle & Thiele, 1996). In general, most of the selection methods present an unsatisfactory performance as a consequence of the deficient relations between elitism and diversity of their selection procedures. To provide an overview of all methods, we summarize the set of comparative features for each method in Table 1.

A selection operator is entirely independent of the Evolutionary Algorithm (EA). In particular, any selection method can be modified or replaced, regardless of the global structure or the rest of the operators used for a specific EA (De la, Maza, & Tidor, 1993). In spite of the importance of the selection operation in the performance of the EAs, the current research for the design or modification of selection operators has been practically overlooked.

The concept of selective pressure (Li et al., 2015; Pascal et al., 2011) has been extensively used in the literature to characterize the performance of a selection approach. The index of takeover time (Goldberg & Deb, 1991) allows evaluating the selective pressure of a determined selection method adequately. It quantifies the number of generations required by a selection method to fill the complete population with copies of the best initial solution. Therefore, initially, the population presents only a single best individual $\mathbf{g}$ ($g_1, g_2, ..., g_d$) and N-1 worse elements. Then, the selection method is operated until the whole population contains N copies of $\mathbf{g}$.

On the other hand, nature is an exciting and inexhaustible source of solutions to several biological problems, which were solved as a result of natural selection during millions of years of evolution (Julian, Olga, Nikolaj, Bowyer, & Pahl, 2006; Sonya & William, 2010). As functional entities, nature has suggested important patterns with interesting characteristics. They have been used as inspiration for a significant number of innovative models that can be extended to solve complex engineering and mathematical problems (Clark, Kok, & Lacroix, 1999). One of the most famous patterns present in nature is the Golden Section (GS) (Benavoli & Chisci, 2009; Newell &Pennybacker, 2013). It defines an especial proportion that allows the adequate formation, selection, partition, and replication of several natural phenomena (Walser, 2001). It appears in a variety of schemes, including the geometry of crystals, the spacing of stems in plants, the proportion of body parts in animals, and in the proportion of feature size in the human face (Dunlap, 1997). GS, sometimes known as the golden ratio or golden number, has been studied widely and has attracted the interest of many scientific communities. As a result, its use has been extended to several disciplines such as architecture (Krishnendra, 2015), arts (Loai, 2012), engineering, industrial design (Lu, 2003), biology (Ciucurel, Georgescu, & Iconaru, 2018) and quantum mechanics (Coldea1 et al., 2010).

Alternatively, to traditional selection methods, in this paper, a new simple selection method for EAs is introduced. In the proposed method, the population is segmented into several groups. The number of individuals and the selection probability of each group is determined so that the proportion of two consecutive groups maintains the GS. The proportions are assigned considering that the group with the highest proportion of individuals corresponds to the smallest probability to be selected. This group assembles the elements with the worst fitness quality of the population. In contrast, the group with the lowest proportion of elements associates the highest probability to be chosen. Such a group contains the best individuals in the population. Since the possibility to choose an element inside the group is the same, the probability to select an individual depends exclusively on the group from which it belongs. Numerical simulations show that the proposed method achieves the best performance over other selection algorithms, with regard to solution quality and convergence speed. The main contributions of this research are: