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Improved solution strategies for dominating trees

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ABSTRACT

Intelligent systems in Wireless Sensor Networks (WSN) allow to eliminate duplicated sensed data and to efficiently route them to a central processing unit. Thus, augmenting WSN lifetime while keeping sensors connectivity and routing the sensed information through an optimized network structure. This work discusses a set of improved exact solution strategies that were effective to the design of a minimum energy consumption structure for the sensors data communication. We present an extended version of a primal-dual model for the minimum cost dominating tree (DT) problem, as well as valid inequalities to improve its linear relaxation. We discuss structural properties of dominating sets and trees used to conceive efficient cutting plane strategies. We adapt accordingly reduction techniques from dominating sets to the DT problem. The new solution techniques allow to handle to optimality challenging benchmark and new randomly generated DT instances with up to 100 nodes, which was not possible to achieve with state-of-the-art models for the problem.

1. Introduction

Wireless sensor networks (WSN) are very important in many applications as monitoring natural disasters, health care devices, area intrusion, among others. Sensed data can contain duplicated information if two or more sensors detect the same event. Sending duplicated data from sensors to a central processing unit leads to an excessive energy consumption, as well as if the sensed data is not routed through a path with minimum energy consumption. Optimizing WSN lifetime is a very promising area of interest in computing and communication technology with the use of unified intelligent softwares (Hacioglu, Kand, & Sesli, 2016). A key ingredient to reduce energy consumption is related to the network topology and the rule of sensors in a WSN. As nodes have a limited battery autonomy, they are now expected to act as expert systems concerning many aspects related to the data transmission based on local sensors knowledge (Reina, Ciobanu, Toral, & Dobre, 2016). The focus of this work is to provide effective exact solution strategies to the design of a minimum energy consumption topology for the sensors data communication in WSN. In this case, intelligent softwares can be integrated to sensors in order to automatically distinguish and eliminate duplicated sensed information and to route the sensed filtered data through an optimized structure, thus augmenting the WSN lifetime.

In a WSN, a node can communicate directly with its neighbors if its radial transmission range reaches them. Connectivity and energy saving are the key ingredients to achieve this goal (Carle & Simplot-Ryl, 2004). Usually, to keep a wireless sensor network operating as long as possible, we distinguish two connected structures for sensor message communications. In the first structure, we assume fixed transmission radius. When a sensor receives a message for the first time, it retransmits this message to its neighbors. This causes a redundant message retransmission, possibly augmenting the network energy consumption that is estimated based on the number of hops needed to accomplish the sensor communications. In order to reduce the number of hops in message retransmissions and, consequently, the network energy consumption, we are interested in a connected dominating set[1] of sensors of minimum cardinality (Gendron, Lucena, da Cunha, & Simonetti, 2014). An alternative approach, instead of considering the number of hops to accomplish sensor communications, consists in adjusting the transmission range. In this more realistic situation, energy consumption is determined as a function of the distance between sensors. This may require more active sensors in the connected

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[1] A set $D \subset S$ is dominating if for all $s \in S$, or $s \in D$ or s is adjacent to some element of D. A set is connected if there exists a path between any pair of its elements.

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dominating set, but reduces individual sensor energy consumption and augments the network lifetime. In this case, message communications are realized by using a tree topology of minimum total distance spanning a dominating set of sensors (Shin, Shen, & Thai, 2010). The first situation is a particular case of the second one when we consider equal distance for all network connections.

The recent literature on mathematical models and solution approaches to solve to optimality the minimum connected dominating set (MCDS) and the minimum dominating tree (MDT) is resumed to a few works (Adasme, Andrade, Leung, & Lisser, 2016; Gendron et al., 2014; Shin et al., 2010). (Shin et al., 2010) proposed heuristics and an integer programming (IP) model for MDT with an exponential number of sub-tour elimination constraints. Their IP model was able to solve to optimality DT instances with up to 17 nodes. Adasme et al. (2016) proposed two compact models for MDT. One of them is based on an extended version (Andrade, 2014) of the polytope of spanning trees introduced in Adasme, Andrade, Letournel, and Lisser (2015). The other primal-dual compact model in Adasme et al. (2016) is based on Miller–Tucker–Zemlin (MTZ) (Miller, Tucker, & Zemlin, 1960) sub-tour elimination constraints. The compact models in Adasme et al. (2016) were able to handle instances with up to 100 nodes. Nevertheless, the instances in Adasme et al. (2016) proved to be very easy to solve when compared to lower edge-density instances generated according to Sundar and Singh (2013). Concerning MCDS, (Gendron et al., 2014) essentially proposed a Benders decomposition algorithm and a branch-and-cut method both based on connected dominating set properties. One of their models is based on the dominating tree concept because if a dominating set is connected, then there exists a tree spanning its nodes. Thus, minimizing the number of edges in a DT is equivalent to minimize the number of nodes in the corresponding connected dominating set.

In this work we are interested in exact solution approaches to solve to optimality the minimum dominating tree problem. We focus on an improved version of the primal-dual model in Adasme et al. (2016) and discuss structural properties of dominating sets and trees that are used to conceive efficient cutting plane strategies to this problem. We show that some valid properties for MCDS (Gendron et al., 2014) and for dominating sets (Alber, Fellows, & Niedermeier, 2004), when adapted accordingly, are very helpful. Our solution strategies are used to improve the linear relaxation of the primal-dual model in Adasme et al. (2016). They allow to handle to optimality some challenging benchmark DT instances (Chaurasia & Singh, 2016; Sundar & Singh, 2013) and two new sets of randomly generated ones with up to 100 nodes in significantly less computational effort and with a considerably lower number of branch-and-bound nodes.

The paper is organized as follows. In Section 2, we present some existing MDT models and develop the main theory leading to an improved model to efficiently solve this problem with the use of a cutting-plane procedure. Section 3 reports numerical results and Section 4 presents the conclusions of this work.

2. MDT models

2.1. Problem definition and complexity

Given an undirected graph $G=(V,E)$ with a set of nodes V and a set of weighted edges E, a dominating tree $T=(V(T),E(T))$ of nodes $V(T)\subset V$, and edges $E(T)\subset E$, is an acyclic connected graph where $V(T)$ is a dominating set, i.e., every node $v\in V$ or is in $V(T)$ or is adjacent to a node in $V(T)$. The weight of a tree T is defined as $\Sigma_{e\in E(T)}c_e$, where $c_e\in\mathbb{R}_+$ is the weight of an edge $e\in E$. The MDT problem is to determine a dominating tree of minimum weight of G.

This problem is NP-hard. To see this, consider the problem of determining a dominating tree of minimum cost in a graph $G=(V\cup F, E_1\cup E_2)$, with $V=\{1,2,\dots,n\}$, $F=\{n+1,n+2,\dots,2n\}$, $E_1=\{uv\,|\,u\in V, v\in V-u\}$, and $E_2=\{uv,\,|\,u\in V, v=u+n\in F\}$. Let us assume that each edge $uv\in E_1$ has positive cost and that the cost of every edge in E_2 is equal to one. Thus, obtaining a Hamiltonian path of minimum cost in the subgraph induced by V, say $G'=(V,E_1)$, known to be NP-hard, gives an optimal solution to the DT problem.

2.2. Graph concepts and notation

For a directed graph $G=(V,A)$ with a set of nodes V and a set of weighted arcs A, a dominating arborescence $T=(V(T),A(T))$ with nodes $V(T)\subset V$ and arcs $A(T)\subset A$, is an acyclic connected directed graph where $V(T)$ is a dominating set. In any arborescence, there is a root node with no incoming arc and the remaining nodes have exactly one incoming arc.

Notations $E(S)$ and $A(S)$ denote subsets of edges of E and arcs of A, respectively, with both endpoints in $S\subset V$. $A(S, V\backslash S)$ represents the subset of arcs of A with one endpoint in $S\subset V$ and the other one in $V\backslash S$. In an undirected graph, the notation $N(v)=\{u|uv\in E\}$ represents the neighborhood of a node v. For directed graphs, we define the negative neighborhood of a node v as $N^-(v)=\{u|uv\in A\}$, its positive neighborhood as $N^+(v)=\{u|vu\in A\}$, and its neighborhood as $N(v)=N^-(v)\cup N^+(v)$. The closed neighborhood of a node v is defined as $N[v]=N(v)\cup\{v\}$ for both directed and undirected graphs. A subgraph $G[S]=(S,E(S))$ of a graph $G=(V,E)$, where $E(S)=\{uv\in E\,|\,u,v\in S\}$, is the graph induced by the set of nodes $S\subset V$. $G[\{v\}]=(\{v\},\emptyset)$ is called a trivial graph.

A node $v\in V$ (resp. an edge $e\in E$) whose exclusion from $G=(V,E)$ disconnects G in two or more connected components is called a cut node (resp. a cut edge). Given two disjoint sets S_1, $S_2\subset V$, if for all $v\in S_2$ there exists $u\in S_1$ such that $uv\in E$, then S_2 is dominated by S_1; otherwise, S_2 is not dominated by S_1.

Given an undirected graph $G=(V,E)$, a dominating tree T of G is represented by an incidence vector $x\in\{0,\ 1\}^{|E|}$, where $x_{uv}=1$ if edge $uv\in E(T)$ and $x_{uv}=0$, otherwise. We also use x variables to represent an arborescence of a directed graph $G=(V,A)$. In this case, $x\in\{0,\ 1\}^{|A|}$, where $x_{uv}=1$ if arc $uv\in A(T)$, and $x_{uv}=0$, otherwise. We use the same notation for edges and arcs to unify the understanding of the next models. We use binary variables $y_v\in\{0,1\}$, for all $v\in V$, where $y_v=1$ if $v\in V(T)$; and $y_v=0$, otherwise. $G[x]$ represents the support graph induced by the non-null components of x.

2.3. Model of Shin et al. (2010)

Consider $G=(V,E)$ an undirected graph. The model in Shin et al. (2010) is

$$(P)\quad \min \sum_{uv\in E} c_{uv}x_{uv} \tag{1}$$

$$s.t.\quad \sum_{uv\in E} x_{uv} - \sum_{v\in V} y_v = -1, \tag{2}$$

$$\sum_{uv\in E(S)} x_{uv} \le |S|-1,\quad \forall S\subset V, \tag{3}$$

$$y_u + y_v - 2x_{uv} \ge 0,\quad \forall\, uv\in E, \tag{4}$$

$$\sum_{u\in N(v)} y_u \ge 1,\quad \forall\, v\in V, \tag{5}$$

$$x_{uv}\in\{0,1\},\ \forall\, uv\in E,\ y_u\in\{0,1\},\ \forall\, v\in V, \tag{6}$$