Expert Systems With Applications 98 (2018) 43–56

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

Piecewise linear value functions for multi-criteria decision-making

Jafar Rezaei

Faculty of Technology Policy and Management, Delft University of Technology, Delft, The Netherlands

ARTICLE INFO

Article history:
Received 24 August 2017
Revised 1 January 2018
Accepted 2 January 2018
Available online 3 January 2018

Keywords:
Multi-criteria decision-making
MCDM
Decision criteria
Value function
Monotonicity

ABSTRACT

Multi-criteria decision-making (MCDM) concerns selecting, ranking or sorting a set of alternatives which are evaluated with respect to a number of criteria. There are several MCDM methods, the two core elements of which are (i) evaluating the performance of the alternatives with respect to the criteria, (ii) finding the importance (weight) of the criteria. There are several methods to find the weights of the criteria, however, when it comes to the alternative measures with respect to the criteria, usually the existing MCDM methods use simple monotonic linear value functions. Usually an increasing or decreasing linear function is assumed between a criterion level (over its entire range) and its value. This assumption, however, might lead to improper results. This study proposes a family of piecewise value functions which can be used for different decision criteria for different decision problems. Several real-world examples from existing literature are provided to illustrate the applicability of the proposed value functions. A numerical example of supplier selection (including a comparison between simple monotonic linear value functions, piecewise linear value functions, and exponential value functions) shows how considering proper value functions could affect the final results of an MCDM problem.

1. Introduction

Decision theory is primarily concerned with identifying the best decision. In many real-world situations the decision is to select the best alternative(s) from among a set of alternatives considering a set of criteria. This subdivision of decision-making, which has gained enormous attention, due to its practical value, in the past recent is called multi-criteria decision-making (MCDM). More precisely, MCDM concerns problems in which the decision-maker faces m alternatives $(a_1, a_2, \ldots, a_m)$, which should be evaluated with respect to n criteria $(c_1, c_2, \ldots, c_n)$, in order to find the best alternative(s), rank or sort them. In most cases, an additive value function is used to find the overall value of alternative i, U_i, as follows:

$$U_i = \sum_{j=1}^{n} w_j u_{ij}, \tag{1}$$

where u_{ij} is the value of alternative i with respect to criterion j, and w_j shows the importance (weight) of criterion j. In some problems, the decision-maker is able to find u_{ij} from external sources as objective measures, in some other problems, u_{ij} reflects a qualitative evaluation provided by the decision-maker(s), experts or users as subjective measures. Price of a car is an objective criterion while comfort of a car is a subjective one. For objective criteria, we usually use physical quantities, for instance, 'International System of Units' (SI), while for subjective criteria, we do not have such standards, which is why we mostly use pairwise comparison, linguistic variables, or Likert scales in order to evaluate the alternatives with regard to such criteria. In order to find the weights, w_j, the decision-maker might use different tools and methods, from the simplest way, which is assigning weights to the criteria intuitively, to use simple methods like SMART (simple multi-attribute rating technique) (Edwards, 1977), to more structured methods like multiple attribute utility theory (MAUT) (Keeney & Raiffa, 1976), analytic hierarchy process (AHP) (Saaty, 1977), and best worst method (BWM) (Rezaei, 2015, 2016). While these methods are usually called 'multi attribute utility and value theories' (Carrico, Hogan, Dyson, & Athanassopoulos, 1997), there is another class of methods, called outranking methods, like ELECTRE (ELimination and Choice Expressing REality) family (Roy, 1968), PROMETHEE methods (Brans, Mareschal, & Vincke, 1984) which do not necessarily need the weights to select, rank or sort the alternatives. What, however, is in common in these methods is the way they consider the nature of the criteria. That is to say, in the current literature, one of the common assumptions about the criteria (most of the time it is not explicitly mentioned in the literature), is monotonicity.

Definition 1 (Keeney & Raiffa, 1976). Let u represents a value function for criterion X, then u is monotonically increasing if:

$$[x_1 > x_2] \Leftrightarrow [u(x_1) > u(x_2)]. \tag{2}$$

E-mail address: j.rezaei@tudelft.nl

https://doi.org/10.1016/j.eswa.2018.01.004
0957-4174/

Definition 2 (Keeney & Raiffa, 1976). Let u represents a value function for criterion X, then u is monotonically decreasing if:

$$[x_1 > x_2] \Leftrightarrow [u(x_1) < u(x_2)]. \quad (3)$$

A function which is not monotonic is called non-monotonic and may have different shapes. For instance, a value function with the first part increasing and the second part decreasing called non-monotonic, by splitting of which, we have two monotonic functions.

This assumption – monotonicity – however, is an oversimplification in some real-world decision-making problems. Another simplification is the use of simple linear functions over the entire range of a criterion. Considering the two assumptions (monotonicity, linearity), we usually see simple increasing and decreasing linear value functions for the decision criteria in MCDM problems. The literature is full of such applications. For instance, many of the studies reviewed in the following review papers implicitly adopt such assumptions: the MCDM applications in supplier selection (Ho, Xu, & Dey, 2010), in infrastructure management (Kabir, Sadiq, & Tesfamariam, 2014), in sustainable energy planning (Pohekar & Ramachandran, 2004), and in forest management and planning (Ananda & Herath, 2009). While in some studies the use of monotonic and/or linear value function might be logical, their use in some other applications might be unfitting. For instance, Alanne, Salo, Saari, and Gustafsson (2007), for evaluation of residential energy supply systems use monotonic-linear value functions for all the selected evaluation criteria including "global warming potential (kg CO_2 m^{-2} a^{-1})", and "acidification potential (kg SO_2 m^{-2} a^{-1})". Considering a monotonic-linear value function for such criteria implies that the decision-maker accepts any level of such harmful environmental criteria for an energy supply system. However, if the decision-maker does not accept some high levels of such criteria (which seems logical), a piecewise linear function might better represent the preferences of the decision-maker (see the decrease-level value function in the next section).

Some authors have discussed nonlinear monotonic value functions (e.g., exponential value functions by Kirkwood, 1997; Pratt, 1964). Others use qualitative scoring to address the non-monotonicity (Brugha, 2000; Kakeneno & Brugha, 2017; O'Brien & Brugha, 2010). We can also find some forms of eliciting piecewise linear value function in Jacquet-Lagreze and Siskos (2001), and Stewart and Janssen (2013). Some other value function construction or elicitation frameworks can be found in Herrera, Herrera-Viedma, and Verdegay (1996), Lahdelma and Salminen (2012), Mustajoki and Hämäläinen (2000), Stewart and Janssen (2013), and Yager (1988). Although in PROMETHEE we use different types of piecewise functions for pairwise comparisons (Brans, Mareschal, & Vincke, 1984), the functions are not used to evaluate the decision criteria. So, despite some efforts in literature, there is no a library of some standard piecewise linear value functions which can be used in different methods like AHP or BWM. It is also important to note that while in many studies value functions are elicited according to the preference data we have from the decision-maker(s), in MCDM, usually we use the value function as a subjective input. This implies that, in MCDM methods (except a few methods, such as UTA), the value function is not elicited, but an approximation is used. This also suggests that the rich literature on determining and eliciting value functions is not actually helping MCDM methods in this area. In this paper, first, a number of piecewise linear value functions with different shapes are proposed to be considered for the decision criteria. It is then shown, with some real-world examples, how such consideration might change the final results of a decision problem. A comparison between simple monotonic linear value functions, piecewise linear value functions, and exponential value functions is conducted, which shows the effectiveness of the proposed pricewise value functions. This is a significant contribution to this field and it is expected to be widely used by MCDM applications.

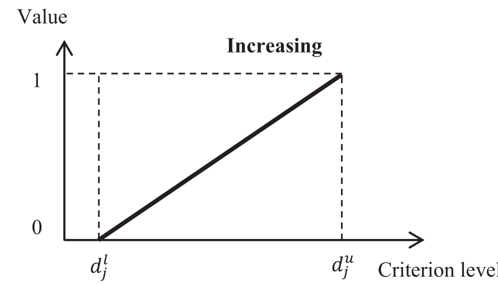

Fig. 1. Increasing value function.

In the next section, some piecewise linear value functions along with some real-world examples are presented, which is followed by some remarks in Section 3. In Section 4, some numerical analyses are used to show the applicability of considering the proposed value functions in a decision problem. In Section 5, the determination of the value functions is discussed. In Section 6, the paper is concluded, some limitations of the study are discussed, and some future research directions are proposed.

2. Piecewise linear value functions

In this section, a number of piecewise value functions are defined for decision criteria. We provide some example cases from the existing literature or practical decision-making problems to support[1] each value function. In all the following value functions we consider $[d_j^l,\ d_j^u]$ as the defined domain for the criterion by the decision-maker; x_{ij} shows the performance of alternative i with respect to criterion j; and u_{ij} shows the value of alternative i with respect to criterion j. For instance, if a decision-maker wants to buy a car considering price as one criterion, if all the alternatives the decision-maker considers are between €17,000 and €25,000, then the criterion might be defined for this range $[17{,}000,\ 25{,}000]$.

2.1. Increasing

Increasing value function is perhaps the most commonly used function in MCDM applications. It basically shows that as the criterion level, x_{ij}, increases, its value, u_{ij}, increases as well. It is shown in Fig. 1 and formulated as follows:

$$u_{ij} = \begin{cases} \dfrac{x_{ij} - d_j^l}{d_j^u - d_j^l}, & d_j^l \le x_{ij} \le d_j^u, \\ 0, & \text{otherwise}. \end{cases} \quad (4)$$

For this function we can think of:

- Product quality in supplier selection (Xia & Wu, 2007). Considering a set of suppliers, a buyer may always prefer a supplier with a higher product quality compared to a supplier with lower product quality.
- Energy efficiency in alternative-fuel bus selection (Tzeng, Lin, & Opricovic, 2005). Considering a set of buses, a bus with more efficient fuel energy might always be preferred to a bus with less efficient fuel energy.

[1] It is worth-mentioning that the studies we discuss to support each value function have some theoretical or practical support for the proposed value functions. It does not, however, mean that those studies have used these value functions in their analysis.