ARTICLE IN PRESS

JID: ESWA [m5G;November 9, 2017;16:27]

Expert Systems With Applications 000 (2017) 1–18

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

A novel two-stage DEA production model with freely distributed initial inputs and shared intermediate outputs

Mohammad Izadikhah[a], Madjid Tavana[b,c,*], Debora Di Caprio[d,e], Francisco J. Santos-Arteaga[f]

[a] Department of Mathematics, College of Science, Arak Branch, Islamic Azad University, Arak, Iran
[b] Business Systems and Analytics Department, Distinguished Chair of Business Analytics, La Salle University, Philadelphia, PA 19141, USA
[c] Business Information Systems Department, Faculty of Business Administration and Economics, University of Paderborn, D-33098 Paderborn, Germany
[d] Department of Mathematics and Statistics, York University, Toronto, M3J 1P3, Canada
[e] Polo Tecnologico IISS G. Galilei, Via Cadorna 14, 39100, Bolzano, Italy
[f] Faculty of Economics and Management, Free University of Bolzano, Bolzano, Italy

ARTICLE INFO

Article history:
Received 21 June 2017
Revised 9 October 2017
Accepted 1 November 2017
Available online xxx

Keywords:
Data envelopment analysis
Intermediate measure
Efficiency
Shared flow
Two-stage process

ABSTRACT

Conventional data envelopment analysis (DEA) models treat the decision-making units (DMUs) as black-boxes: inputs enter the system and outputs exit the system, with no consideration for the intermediate steps characterizing the DMUs. As a result, intermediate measures are lost in the process of changing the inputs to outputs and it becomes difficult, if not impossible, to provide individual DMU managers with specific information on what part of a DMU is responsible for the overall inefficiency. This study defines a two-stage DEA model, where each DMU is composed of two sub-DMUs in series, the intermediate products by the sub-DMU in Stage 1 are partly consumed by the sub-DMU in Stage 2, and the initial inputs of the DMU can be freely allocated in both stages. Also, there are additional inputs directly consumed in Stage 2 while part of the outputs of Stage 1 are final outputs. We develop four new linear models to determine the upper and lower bounds of the efficiencies of the two sub-DMUs in a non-cooperative setting and a linear model to calculate the overall efficiency of DMU in a cooperative setting. That is, the overall efficiency of a DMU is modelled in a cooperative setting via upper and lower bounds obtained in the non-cooperative one. The proposed two-stage DEA method allows for important applications to several management areas. A case study in the banking industry is presented to demonstrate the applicability and exhibit the efficacy of the proposed models.

1. Introduction

Data envelopment analysis (DEA) is an effective non-parametric evaluation method for measuring the relative efficiency of a set of decision making units (DMUs) each of which uses multiple inputs to produce multiple outputs. In the traditional DEA methods, DMUs are treated as black-boxes, that is, the internal structure of the DMUs is often ignored. As a result, the focus of the investigation is on the single operational processes with a set of initial inputs and final outputs that is unable of pinpointing the sources of inefficiency within the DMUs (Lewis, Mallikarjun, & Sexton, 2013; Wang, Huang, Wu, & Liu, 2014). On the other hand, in the two-stage DEA models, DMUs are modelled as systems composed of two sub-DMUs in series where the outputs of the sub-DMU in Stage 1, known as intermediate measures/products/outputs, are considered as inputs of the sub-DMU in Stage 2. As a result, a two-stage DEA model allows one to further investigate the structure of a DMU and its processes and, hence, to identify the misallocation of the inputs among the sub-DMUs (Du, Liang, Chen, Cook, & Zhu, 2011; Ebrahimnejad, Tavana, Lotfi, Shahverdi, & Yousefpour, 2014; Yu, Shi, & Song, 2013).

Despite appearing as the simplest multi-stage approach to efficiency evaluation, two-stage DEA models are the building blocks for the study of series systems whose DMUs consist of multiple sub-DMUs operating through procedures of different complexity. The work of Charnes et al. (1986) on army recruitment using a two-stage approach was the first study to discuss the loss of information intrinsic to single-stage models. In recent years, many researchers have studied several applications of two-stage DEA models to diverse decision making situations such as healthcare man-

* Corresponding author at: Business Systems and Analytics Department, Distinguished Chair of Business Analytics, La Salle University, Philadelphia, PA 19141, USA.
E-mail addresses: m-izadikhah@iau-arak.ac.ir (M. Izadikhah), tavana@lasalle.edu (M. Tavana), dicaper@mathstat.yorku.ca, debora.dicaprio@istruzione.it (D. Di Caprio), fsantosarteaga@unibz.it (F.J. Santos-Arteaga).

https://doi.org/10.1016/j.eswa.2017.11.005
0957-4174/

agement programs (Schinnar, Kamis-Gould, Delucia, & Rothbard, 1990), education programs (Lovell, Walters, & Wood, 1994), and sport teams (Sexton & Lewis, 2003), among others. Among the many possible applications, those to the banking industry have been attracting a considerable amount of attention (Seiford & Zhu, 1999; Ebrahimnejad et al., 2014). More in general, Castelli, Pesenti, and Ukovich (2010) provide a comprehensive categorized overview of the models and methods that have been developed for different multi-stage production architectures.

The problem of evaluating the efficiency of DMUs can also be considered from the game theoretical perspective. In this sense, there are two main approaches to the problem: one approach models two-stage processes as non-cooperative games, the other as centralized systems aiming at maximizing their overall efficiency (Li, 2017). One of the first studies to employ the concepts of cooperative and non-cooperative games in an efficiency evaluation context referring to two-stage DMUs is due to Liang, Yang, Cook, and Zhu (2006). In the non-cooperative setting, one of the two stages is considered more important than the other: the former is identified with *the leader* while the latter with *the follower*. As for the efficiency scores, one starts by measuring and optimizing the leader's efficiency without considering the follower; hence, the follower's efficiency is measured under the constraint that the leader's efficiency remains constant. In the cooperative setting, both stages have the same importance and their efficiency scores must be measured and optimized at the same time (Hosseinzadeh Lotfi, Jahanshahloo, Hemati, & Givehchi, 2012; Mahdiloo et al., 2016). For example, interpreting the offices of a regional R&D program as two-stage DMUs whose first stage consists of Premium Acquisition and the second stage is Profit Generation, we could assume that the Stage 1 is the leader. Thus, the efficiency of Stage 2 would be computed subject to the constraint that the efficiency of Stage 1 remains fixed. Alternatively, we could assume that Stage 2 is the leader and, hence, measure the efficiency of Stage 1 keeping the efficiency of Stage 2 constant (Li, Chen, Liang, & Xie, 2012; Yu & Shi, 2014).

Kao and Hwang (2008) presented a two-stage DEA model where the efficiency of the entire process can be decomposed into the product of the efficiencies of its two sub-processes. However, their approach has a few limitations (Guo, Abbasi Shureshjani, Foroughi, & Zhu, 2017). To start, it can be applied only when the assumption of constant returns to scale is satisfied. Moreover, it allows to model only *closed* multi-stage processes, that is, processes where the outputs of a stage are entirely used as inputs in the following stage with no extra input added. If, for instance, additional inputs are introduced from one stage to the following one, the resulting model becomes non-linear in the multiplicative approach. This is one of the main drawbacks associated with two-stage structures. Zha and Liang (2010) slightly improved the situation by presenting a method for studying a two-stage production process in series where the initial inputs can be freely allocated in the two stages. However, their method yields a parametric model that has other pitfalls. In particular, it does not allow for additional direct inputs to be used in Stage 2 or the existence of shared intermediate products. In order to improve both the method of Zha and Liang (2010) and the structure of two-stage systems, Yu and Shi (2014) proposed a two-stage DEA model where additional inputs are allocated in Stage 2 and part of the outputs of Stage 1 are used as inputs in Stage 2. However, they formulated a parametric model to solve their two-stage structure that does not allow for shared inputs or final outputs to be produced directly in Stage 1. The method proposed in this study deals with the aforementioned problems providing a viable solution.

In this paper, we consider a novel two-stage DEA model, where each DMU is composed of two sub-DMUs in series, the intermediate products of the sub-DMU in Stage 1 are partly consumed by the sub-DMU in Stage 2, and the initial inputs of the DMU can be freely allocated to either one of the two sub-DMUs. Also, there are additional inputs directly consumed in Stage 2 while the sub-DMU in Stage 1 is allowed to produce final outputs.

Building on the concept of Stackelberg leader–follower game and the work of Yu and Shi (2014), we formulate suitable fractional programming problems that allow to determine the upper and lower bounds of the efficiencies of the two sub-DMUs in the non-cooperative setting. Afterwards, we consider the cooperative setting and define a fractional programming model to calculate the overall efficiency of DMUs. All the models are linearized by applying a Charnes–Cooper transformation (Charnes & Cooper, 1962).

To show the advantages of using the proposed method in place of the existing ones, we provide some numerical comparisons with the methods defined by Kao and Hwang (2008), Zha and Liang (2010) and Yu and Shi (2014). These comparisons show that our method is much more general than the existing models in terms of closeness and consistency of the solutions.

According to Zha and Liang (2010), Matthews (2013), and Akther, Fukuyama, and Weber (2013), a banking system can be viewed as a two-stage process. Moreover, following Seiford and Zhu (1999), Zhu (2000), Luo (2003), and Liu (2011b), the first stage can be used to measure the *profitability* of a financial company (its ability to generate revenue and profit using its labor, assets, and capital resources) while the second stage measures its *marketability* (the ability of the company to generate revenue and profit through its stock market performance). We will modify the two-stage evaluation framework proposed by Seiford and Zhu (1999) and Liu (2011b) and adapt it to analyze 15 branches of a commercial bank in the Philadelphia metropolitan area. The results obtained by using our intermediate-like approach are compared with those delivered by a classical black-box approach showing that the latter approach does not suffice to adequately evaluate the aggregate performances of the DMUs composing a system.

The paper unfolds as follows. Section 2 presents a literature review while Section 3 provides a detailed review of three specific two-stage DEA models that will be used as reference points to define our approach. Section 4 extends Yu and Shi (2014)'s method to the proposed two-stage DEA production model with freely distributed inputs and shared intermediate outputs. Section 5 provides some numerical comparisons among the proposed method and those defined by Kao and Hwang (2008), Zha and Liang (2010) and Yu and Shi (2014) showing the advantages of our model over the existing ones. Section 6 describes a case study in the US banking industry and discusses the corresponding numerical results. Finally, Section 7 concludes and suggests some future research directions.

2. Literature review

In this section, we review some the vast literature related to the present study.

2.1. A general review of two-stage DEA models

2.1.1. A glance at some early applications of two-stage DEA

Färe and Whittaker (1995) and Färe and Grosskopf (1996) applied an input oriented two-stage network DEA model to study relative efficiencies in dairy production processes. Wang, Gopal, and Zionts (1997) and Noulas, Hatzigayios, Lazaridis, and Lyroudi (2001) proposed applications of two-stage DEA to information technology (IT) and to non-life insurance policies, respectively. Färe and Grosskopf (2000) presented a network DEA model for the Swedish Institute for Health Economics. Seiford and Zhu (1999) divided a complicated production process into independent sub-processes and, hence, calculated the efficiencies of