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Modeling and solving the non-smooth arc routing problem with realistic soft constraints☆

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ABSTRACT

This paper considers the non-smooth arc routing problem (NS-ARP) with soft constraints in order to capture in more perceptive way realistic constraints violations arising in transportation and logistics. To appropriately solve this problem, a biased-randomized procedure with iterated local search (BRILS) and a mathematical model for this ARP variant is proposed. An extensive computational study is conducted on rich and diverse problem instances. The results highlight the competitiveness of BRILS in terms of quality and time, where it provides high-quality solutions within reasonable computational times. In the context of real-world environments, the performance exhibited by BRILS motivates its incorporation in intelligent and integrative systems where frequent and fast solutions are required.

1. Introduction

In real-life logistics and transportation, many decision-making processes can be modeled as combinatorial optimization problems (OPs) (Montoya-Torres, Juan, Huaccho Huatuco, Faulin, & Rodriguez-Verjan, 2012). In a combinatorial OP, a large number of feasible solutions need to be explored in order to select one that minimizes cost, maximizes benefit, etc. Frequently, combinatorial OPs have a well-structured definition consisting of an objective function to be optimized and a set of hard constraints that need to be satisfied (Papadimitriou & Steiglitz, 1982). These OPs can usually be formulated using mixed integer linear programming (MILP) models. However, most of them are NP-hard in nature, which implies that only small- to medium-sized instances can be solved in reasonable computing times by means of classical MILP methods (Garey & Johnson, 1990). Thus, heuristics and metaheuristics are usually required to solve medium- to large-sized instances. An additional difficulty arises when the mathematical model of the problem does not meet certain characteristics. In effect, when properties such as convexity and smoothness are not fulfilled, the solution space might become highly irregular. In such situations, finding an optimal or near-optimal solution becomes a challenging task even for medium-sized instances.

In this work, we study a relevant combinatorial OP under a non-smooth environment. In particular, we focus on the arc routing problem (ARP) with realistic soft constraints, which impose penalty costs on the objective function. These penalty costs are usually defined as piecewise functions, thus transforming the objective function into a non-smooth one. The ARP is a combinatorial OP originally introduced by Golden and Wong (1981). In its basic version, the ARP is composed of a central depot, a fleet of identical vehicles, and a network of arcs connecting nodes. Some of these arcs have a demand that must be served (see Fig. 1). Also, there is a cost associated with traversing each of the arcs. Under these circumstances, the usual goal is to find a set of routes (solution) which minimizes the total delivering cost while satisfying the following constraints: *(i)* every route starts and ends at the depot, *(ii)* each arc with a positive demand is served exactly by one vehicle, and *(iii)* the total demand to be served in any route does not exceed the vehicle loading capacity. The main novelty we introduce to this basic version is the inclusion of soft constraints, i.e., we consider a non-smooth penalty term to the objective function to penalize every time a given constraint is not fully satisfied. As discussed in Hashimoto, Ibaraki, Imahori, and Yagiura (2006), *"in real-world simulations, time windows and capacity constraints can be often violated to some extent"*. In this paper thus, a threshold is im-

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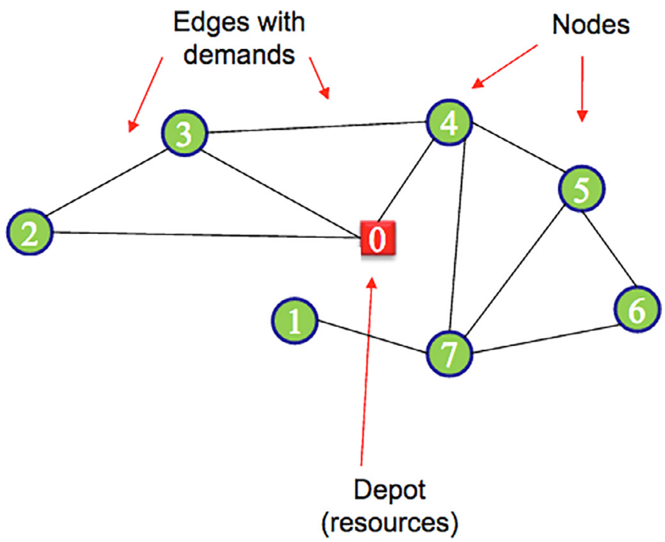

Fig. 1. Illustration of the arc routing problem.

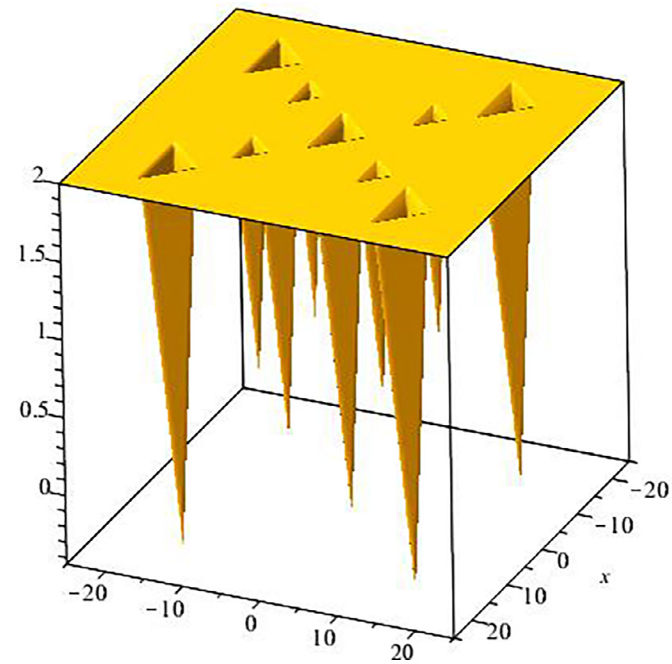

Fig. 2. Example of a non-convex and non-smooth objective function.

posed on the maximum cost (distance-based or time-based) any given route can take. Then, whenever a route exceeds this threshold a penalty cost is added to the total route cost. As in many real-life applications, this penalty cost will depend on the size of the gap between the actual route cost and the threshold.

Accordingly, the main contributions of this paper are: *(i)* a discussion on the importance of considering non-smooth objective functions in realistic combinatorial OPs, mainly due to the existence of soft constraints; *(ii)* an original mathematical model that formally describes the non-smooth ARP (NS-ARP); *(iii)* a metaheuristic approach, based on the integration of biased-randomized techniques (Grasas, Juan, Faulin, de Armas, & Ramalhinho, 2017) inside an iterated local search framework (Lourenço, Martin, & Stützle, 2010), to efficiently cope with the NS-ARP; and *(iv)* the use of CPLEX to solve a truncated version of the NS-ARP model, which allows us to obtain lower bounds for assessing our metaheuristic approach.

Our current work contributes to the field of Expert and Intelligent Systems in several ways. Firstly, it considers of a more realistic ARP incorporating penalty costs when certain constraints are violated, i.e., introducing soft constraints. This is a more intelligent way to address constraints violations, as we demonstrate comparing to other perspectives. Additionally, as mentioned before, we propose a solving methodology combining biased randomized techniques with an iterated local search metaheuristic framework. This system leads to important savings for the related logistics and transportation industry, and a mathematical model for this non-smooth version of the ARP help us to confirm its goodness. In the context of expert systems, several works have analyzed non-smooth optimization problems. Thus, for instance, Roy, Ghoshal, and Thakur (2010) deal with non-smooth power-flow problems, Lu, Zhou, Qin, Li, and Zhang (2010) propose an adaptive hybrid differential evolution algorithm to cope with a non-smooth version of the dynamic economic dispatch problem, and Ferrer, Guimarans, Ramalhinho, and Juan (2016) propose a biased-randomized metaheuristic to solve the non-smooth flow-shop problem. As far as we know, however, there is a lack of publications considering realistic non-smooth cost functions in routing problems, which enhances the importance of this expert system.

The remaining of the paper is structured as follows: Section 2 reviews some basic concepts related to non-convex / non-smooth optimization problems, as well as how metaheuristics have been used in solving them. Section 3 presents a survey on recent works on the ARP. A mathematical description of the NS-ARP is provided in Section 4. In Section 5 a biased-randomized iterated local search algorithm is proposed as a solving method for the NS-ARP. Section 6 is devoted to explaining the computational experiments performed. Finally, conclusions and future work are described in Section 7.

2. Metaheuristics in non-convex / non-smooth OPs

Optimization problems can be classified as either convex or non-convex. In general, convex OPs have two parts: a series of constraints that represent convex regions and an objective function that is also convex. Linear programming (LP) problems represent a well-known example of convex OPs since linear functions are trivially convex. Other examples of convex OPs include quadratic programming, geometric programming, conic optimization, or least squares (Boyd & Vandenberghe, 2004). In a convex OP, every constraint restricts the space of solutions to a certain convex region. By taking the intersection of all these regions we obtain the set of feasible solutions, which is also convex. Due to the structure of the solution space, every single local optimum is a global optimum too. This is the key property that allows to efficiently solve convex OPs up to very large instances. Unfortunately, the algorithms applied for solving convex OPs cannot be easily extended to solve the non-convex case. The solution space of the non-convex OPs is far more complex since the objective function and/or the region of feasible solutions are not convex. As a consequence, there exist many disjoint regions and multiple locally optimal points within each of them (Fig. 2). Thus, a traditional local search is not enough since there is a risk of ending in a local optimum that may still be far from the global one. In addition, it can take an unreasonable amount of time to conclude that a non-convex OP is unfeasible, or that the objective function is unbounded, or even that the best solution found so far is, in fact, the global optimum.

Non-smooth OPs are similar to non-convex OPs in the sense that they are much more difficult to solve than traditional smooth and convex problems. A function is smooth if it is differentiable and it has continuous derivatives of all orders. Therefore, a non-smooth function is one that has missed some of these proper-