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Allocating a fixed cost based on a DEA-game cross efficiency approach

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ABSTRACT

In many real managerial applications, an issue of considerable importance is allocating a total fixed cost across a set of competing decision making units (DMUs). The fixed cost allocation problem has also become one of the most important applications of the data envelopment analysis (DEA) methodology. In this paper, we will approach the fixed cost allocation problem by explicitly considering both competition and cooperation relationships among DMUs. To this end, we integrate cooperative game theory and the cross efficiency method to develop a DEA-game cross efficiency approach to generate a unique and fair allocation plan. With the proposed approach, each DMU is considered as a player and a super-additive characteristic function is defined for coalitions of DMUs. Then, the Shapley value is calculated for each DMU and accordingly associated common weights are optimized to determine the final allocation plan. Since the cross efficiency method considers peer appraisal and the cooperative game theory allows for equitable negotiations, all DMUs are supposed to reach a consensus on the equitable allocation scheme through our novel approach. From this perspective, our proposed approach is promising and attractive for allocating a fixed cost in large organizations. Finally, the DEA-game cross efficiency approach is demonstrated with a numerical example derived from previous literature and the results are compared to some existing methods. Additionally, we apply the proposed approach to an empirical application concerning city commercial bank activities in China.

1. Introduction

Data envelopment analysis (DEA) is a classic data-based mathematical programming approach for relative efficiency evaluation. The entities to be evaluated are formally called decision making units (DMUs), each of which consumes multiple inputs to produce multiple outputs. DEA was first introduced by Charnes, Cooper, and Rhodes (1978), who expressed the relative efficiency as a ratio of aggregated outputs to aggregated inputs. The basic idea behind the DEA methodology is that a convex combination of a set of comparable and homogeneous DMUs is calculated to construct an efficient frontier. Then each DMU can be projected onto the frontier and the DMU is evaluated by comparing its actual performance to its projection. Because of its various advantages, mainly its flexibility in terms of weights determination and the nonparametric property of specifying production functions (Lin, Lee, & Chiu, 2009), the DEA methodology has been applied to many activities in many situations (Adler, Friedman, & Sinuany-Stern, 2002; Emrouznejad, Parker, & Tavares, 2008; Cook & Seiford, 2009; Liu, Lu, & Lu, 2016; Li, Zhu, & Zhuang, 2017).

In many real applications, the managers of large organizations frequently face the problem of allocating a total fixed cost or input resources to its branches. In theory, the allocation scheme should be in line with the causation principle. However, particularly when assigning fixed costs, we often have the problem that their causes cannot be determined exactly. Thus, companies mostly use size or activity-related distribution criteria to get a reasonable approximation. The fixed cost allocation problem has now become one of the most important application areas of the data envelopment analysis (DEA) methodology (Cook & Kress, 1999; Beasley, 2003; Cook & Zhu, 2005; Li, Yang, Liang, & Hua, 2009; Amirteimoori & Tabar, 2010; Lin, 2011a, 2011b; Li, Yang, Chen, Dai, & Liang, 2013; Du, Cook, Liang, & Zhu, 2014; Jahanshahloo, Sadeghi, & Khodabakhshi, 2017; Li, Song, Dolgui, & Liang, 2017). Typical examples of fixed costs are the advertising expenditure of a manufacturer across its retailers (Cook & Kress, 1999) and the cost of a common communication cable among its users (Beasley, 2003).

The first DEA-based fixed cost allocation approach was made by Cook and Kress (1999). In that seminal work, two basic prin-

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ciples, namely, efficiency-invariance and input Pareto-minimality, were used such that the allocation plan will not affect the relative efficiencies and input transfer is impossible without changing the DEA efficiency scores. The Cook and Kress (1999) approach was also implicitly based on the proportional sharing principle, but it was initially proposed to examine whether these principles have been satisfied, not to generate allocation plans that can satisfy these principles. As the authors themselves indicated, their approach “cannot be used to determine a cost allocation among the DMUs”. In subsequent work, Cook and Zhu (2005) extended the Cook and Kress (1999) approach to “provide a practical approach wherein cost allocations can actually be achieved”. Jahanshahloo, Lotfi, Shoja, and Sanei (2004) argued that the Pareto-minimality principle was invalid based on a numerical example. Lin (2011a) proved that the Cook and Zhu (2005) method would be infeasible when some special constraints were added. Jahanshahloo et al. (2004), Amirteimoori and Kordrostami (2005), Lin (2011a, 2011b), Jahanshahloo et al. (2017) and Li, Song et al. (2017) worked further to generate a final allocation plan in such a way that the efficiencies are based on a common set of weights that are unchanged before and after the fixed cost allocation.

On the other hand, Beasley (2003) suggested a different research stream, in which the fixed cost is allocated in such a way that the DEA efficiency scores are maximized. Notably, Beasley (2003) proposed a series of nonlinear models to maximize the average efficiency across all DMUs, and the final allocation plan is obtained by minimizing the variation to the minimum allocated costs for all DMUs. Amirteimoori and Kordrostami (2005) argued, based on some numerical results, that the Beasley (2003) method will create no feasible solutions in many cases, but recently Jahanshahloo et al. (2017) formally demonstrated that the Beasley (2003) method is always feasible. Both Beasley (2003) and Amirteimoori and Tabar (2010) believed that all DMUs would have an efficiency score of one based on a feasible allocation scheme and a common set of relative weights, and this idea was mathematically proved by Li et al. (2013) and Si et al. (2013). Since multiple allocation plans can make all DMUs efficient, Li et al. (2013) determined a unique allocation plan by maximizing the satisfaction degree for all DMUs. Similar work can also be found in Amirteimoori and Tabar (2010) and Hosseinzadeh Lotfi, Hatami-Marbini, Agrell, Aghayi, and Gholami (2013), where a set of common weights was used to guarantee a series of 100% post-allocation efficiencies. Du et al. (2014) proposed a cross efficiency iterative algorithm as an approach to the fixed cost and resource allocation problem. The final allocation plan is generated in such a way that the post-allocation cross efficiency is maximized for all DMUs. All previous studies considered the allocated costs as additional inputs, whereas Li et al. (2009) considered a special case where other cost measures exist, so the allocated cost was taken as a complement to existing measures. Yu, Chen, and Hsiao (2016) and Zhu, Zhang, and Wang (2017) extended the fixed cost allocation problem to network situations by considering internal two-stage processes. Both Yu et al. (2016) and Zhu et al. (2017) try to maximize the post-allocation efficiencies, but not all DMUs will be efficient after affording the allocated costs.

To determine a fair and unique allocation plan that is acceptable to all DMUs, there is no doubt that the DMUs will compete and cooperate with each other simultaneously. On the one hand, each DMU is selfish and will compete to be allocated fewer costs. On the other hand, all DMUs will cooperate to reach a consensus and to generate the final allocation scheme. However, among the existing DEA-based fixed allocation approaches, few have taken the cooperative game relationship into account. Nakabayashi and Tone (2006) suggested a DEA game to address the egoist’s dilemma when multiple units have conflicts in reaching a consensus for multiple criteria evaluation. Their basic framework can be used for fixed cost allocation and benefit-cost distributions. Yang and Zhang (2015) used a modified Shapley value and Gini coefficient to allocate the fixed cost, in which the DEA efficiency scores are considered as the DMUs’ contribution to coalitions.

By surveying the previous literature, we find that almost all DEA-based fixed cost allocation approaches are primarily based on the efficiency principle, which can be categorized into efficiency invariance and efficiency maximization (Cook & Kress, 1999; Beasley, 2003). For an overview of existing DEA-based fixed cost allocation approaches, we summarize some DEA-based fixed cost allocation approaches in Table 1, which implies a possible research gap. We can learn from Table 1 that: (1) it is important to generate a unique allocation plan that will assign a positive cost responsibility to each DMU, but some existing methods fail to do this. Further, (2) the work based on a game-DEA approach is rare, although it is acknowledged that there exist both competition and cooperation relationships in allocating the fixed cost (Du et al., 2014; Nakabayashi & Tone, 2006; Yang and Zhang, 2015). We believe that the game-based approach will prove significant in allocating the total fixed cost across multiple peer DMUs. Additionally, (3) most researchers are aware of the importance of peer appraisal in reaching a consensus on the allocation plan and thus adopt peer appraisal. However, almost all existing approaches consider peer appraisal by using common weights except for Du et al. (2014) and Yu et al. (2016), but it is difficult for DMUs to necessarily determine a common set of weights in the DEA framework. More importantly, both the consideration of competition and cooperation relationship and peer appraisal are very important and promising for solving the fixed cost allocation problem, but no work has integrated these two considerations to fulfill this research gap.

By investigating the fixed cost allocation problem thoroughly, we find that the main concern is that all DMUs should accept the allocation result. For this purpose, the peer appraisal is an important mechanism. Among all extensions of DEA methodology, the cross efficiency method suggested by Sexton, Silkman, and Hogan (1986) is an excellent example of peer appraisal. In traditional DEA models, each DMU will choose a set of relative weights to maximize its efficiency score to the greatest extent. In the cross efficiency method, however, a DMU’s relative efficiency is measured by using peer DMUs’ preferred weights. Thus, the cross efficiency evaluation uses peer appraisal instead of self-evaluation, and the results are more acceptable. The cross efficiency evaluation not only provides a full ranking of all DMUs, but also eliminates unrealistic weight schemes without any weight restrictions (Wu, Liang, Zha, & Yang, 2009), and thus the cross efficiency method has been comprehensively extended and applied to many areas. For example, Wu, Sun, Liang, and Zha (2011) used Shannon entropy to determine weights for ultimate cross efficiency. Wang and Chin (2010) proposed a neutral DEA model which is neither aggressive nor benevolent. Wang, Chin, and Luo (2011) added a virtual ideal and a virtual anti-ideal DMU into the sample, and the relative weights are determined regarding the distance to the ideal or anti-ideal DMU. Wu, Liang et al. (2009) and Washio and Yamada (2013) incorporated rank preferences for cross efficiency evaluation. Wu, Chu, Sun, and Zhu (2016) proposed a cross efficiency evaluation approach based on Pareto improvement. Wu, Liang, and Yang (2009) used a cross efficiency method to assess the national performance in the Summer Olympic Games. Yu, Ting, and Chen (2010) studied many information-sharing scenarios in supply chains and used cross efficiency models to evaluate the supply chain performance. Falagario, Sciancalepore, Costantino, and Pietroforte (2012) proposed a cross efficiency approach to address the supplier selection problem in public procurement, in which the cross efficiency evaluation guarantees a fair and equal evaluation of all bids. Lim, Oh, and Zhu (2014) pro-