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Existence results for extremal solutions of interval fractional functional integro-differential equations

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Abstract

In this study, we introduce interval fractional functional integro-differential equations (IFFIDEs) under the Caputo generalized Hukuhara fractional differentiability. We establish some necessary comparison results and use the monotone iterative technique combined with the method of upper and lower solutions to investigate the existence of extremal solutions for IFFIDEs. An example is presented to illustrate the results.

Keywords: Extremal solution; Fractional interval-valued differential equation; Fractional interval-valued functional integro-differential equation; Method of upper and lower solutions

1. Introduction

During the formulation of mathematical descriptions of many phenomena related to scientific research, functional differential equations and systems are important tools for achieving an adequate explanation and for accurate adjustment to the behavior of the particular magnitude of interest. These tools are used in fields such as biology, engineering, physics, and other sciences. Many studies have described functional differential equations and their applications, such as monographs by [13,20,21], and the references therein. In addition, fractional differential equations have been investigated widely due to their applications in various fields of engineering and science. Some basic information and results for various types of fractional differential equations can found in the monographs by Samko et al. [34], Podlubny [36], and Kilbas et al. [19].

Recently, Agarwal et al. [1,2] advanced the study of fuzzy fractional differential equations, where they formulated Riemann–Liouville differentiability based on the Hukuhara differentiability as the basis for defining the concept of fuzzy fractional differential equations. Subsequently, they proved the existence of solutions for fuzzy fractional inte-

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gral equations under compactness type conditions by using the Hausdorff measure of non-compactness. Arshad and Lupulescu [6] proved some results based on the existence and uniqueness of solutions to fuzzy fractional differential equations under Hukuhara fractional Riemann–Liouville differentiability. Allahviranloo et al. [3] proposed the concept of the generalized Hukuhara fractional Riemann–Liouville differentiability of fuzzy-valued functions. Subsequent studies investigated the explicit solutions of fuzzy fractional differential equations under Riemann–Liouville H-differentiability. Salahshour et al. [38] provided some new results regarding the existence and uniqueness of solutions of fuzzy fractional differential equations by introducing the fuzzy Laplace transforms. In addition, the fuzzy Laplace transforms of the Caputo Hukuhara derivative were introduced to solve the Basset problem [39]. Mazandarani and Kamyad [28] first studied the numerical solution of the fuzzy fractional initial value problem under Caputo-type fuzzy fractional derivatives by using a modified fractional Euler method. Studies have also provided some results regarding the existence and uniqueness of solutions to fuzzy fractional differential equations under the Caputo type-2 fuzzy fractional derivative as well as giving definitions of the Laplace transforms of type-2 fuzzy number-valued functions [29]. According to the concept of Caputo-type fuzzy fractional derivative in the sense of the generalized fuzzy differentiability, Fard et al. [12] extended and established definitions based on fuzzy fractional calculus of the variation and provided some necessary conditions for obtaining the fuzzy fractional Euler–Lagrange equation for both constrained and unconstrained fuzzy fractional variational problems. Malinowski [26] presented mathematical foundations for studies of random fuzzy fractional integral equations that involve a fuzzy integral of fractional order. Hoa [14,15] provided some existence and uniqueness results for solutions to fuzzy fractional differential equations with delay, and the modified fractional Euler method was investigated for problems of this form.

The connection between fuzzy analysis and interval analysis is well known (Moore and Lodwick [31], Pedrycz and Gomide [35]). Interval analysis and fuzzy analysis were introduced in order to handle the interval uncertainty that appears in many mathematical or computer models of some deterministic real-world phenomena. In addition, interval-valued differential equations are suitable tools for modeling dynamic systems with uncertainties or vagueness. This theory has been developed in several theoretical directions, and a large number of applications have been considered for many different real problems (e.g., see [16,24,25,40,41]). Recently, Lupulescu [17] used a generalization of the Hukuhara difference for a closed interval on the real line in order to develop a theory of the fractional calculus for interval-valued functions. In addition, the Riemann–Liouville fractional integral, Riemann–Liouville fractional derivative, and Caputo fractional derivative were proposed for interval-valued functions. These results provide powerful tools for studying interval-valued fractional differential equations and fuzzy fractional differential equations.

The monotone iterative technique with the method of upper and lower solutions is an effective tool for obtaining the existence result in a closed set generated by the lower and upper solutions. According to this method, if we can find a lower solution X^L and an upper solution X^U of an interval fractional functional integro-differential equation (IFFIDE), and if $X^L \preceq X^U$, then a solution exists that satisfies $X^L \preceq X \preceq X^U$. Many studies have investigated this technique and various nonlinear problems have been addressed. A comprehensive overview of this technique was provided by [22]. Continuous development has occurred in this area and some recent studies have addressed various problems, such as those by [5,7,11,33]. In this study, we consider an initial value problem for IFFIDEs and we use several tools from interval calculus to approximate their extremal solutions in a given interval functional interval by employing the method of upper and lower solutions as well as the monotone iterative technique. To develop the monotone method, we establish some properties related to order and convergence in the interval space, and some necessary comparison theorems, which we use to obtain our main result. In brief, our aims are as follows.

1. We show the equivalence of the IFFIDE and interval fractional functional integral equation under suitable conditions.
2. We prove the existence of a solution for an IFFIDE by using the method of upper and lower solutions.
3. We develop a monotone iterative technique to obtain the existence results for the maximal and minimal solutions of IFFIDEs.

The remainder of this paper is organized as follows. In Section 2, we recall some basic concepts and notations related to interval analysis and the integral differential calculus for interval-valued functions. Moreover, some necessary comparison theorems are established. In Section 3, we use the method of upper and lower solutions and the monotone iterative technique to prove that the maximal and minimal solutions exist, and then under more conditions, we present the uniqueness result for the solution of an IFFIDE. Finally, we give an example to illustrate the theory.

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