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On a system of fuzzy fractional differential inclusions with projection operators ☆

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Abstract

The aim of this paper is to introduce and study a class of fuzzy fractional differential inclusions with projection operators, which captures the desired features of both fuzzy differential inclusions and fractional-order projective dynamical systems within the same framework. The existence of solutions for the open situation is proved by using continuous selection theorem. Moreover, the existence of solutions for the closed situation is showed by employing Lipschitzean selection theorem and fixed point theorem, respectively. Furthermore, an application and a numerical example are also given.

Keywords: Fuzzy fractional differential inclusions; Projection operator; Selection theorem; Fixed point theorem

1. Introduction

Fuzzy differential equations (inclusions) are becoming increasingly important due to their application for modelling uncertain phenomena in different fields, such as biology, chemistry, epidemiology, physics, mechanics, operations research, microelectronics, economics, and finance. In recent years, much effort has been made in this interesting field (see, for instance, [8,12,26,31,36,37,45] and references therein). In particular, in 1997, Hüllermeier [24] was motivated by application in knowledge-based systems, then he proposed a class of fuzzy differential equation based on a family of differential inclusion

$$\begin{cases} x'(t) \in [F_{(t,x(t))}]_\alpha, \quad \alpha \in [0,1], \\ x(0) \in [x_0]_\alpha. \end{cases}$$

This method was exploited by Diamond (see, e.g. [16,17]). In 2000, Zhu and Rao [56] considered the following differential inclusions for fuzzy mappings

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$$\begin{cases} F_{(t,x(t))}(x'(t)) > (\text{or } \geq)\, \alpha(x(t)), \\ \text{i.e., } x'(t) \in \left(F_{(t,x(t))}\right)_{\alpha(x(t)} \left(\text{or } [F_{(t,x(t))}]_{\alpha(x(t))}\right), \\ x(t_0) = x_0. \end{cases} \tag{1.1}$$

Recently, Min et al. [39–41] extended this type of fuzzy differential inclusions to fuzzy delay differential inclusions, two related fuzzy differential inclusions and implicit fuzzy differential inclusions. They obtained some new existence theorems in connection with the solutions of the fuzzy delay differential inclusions and also provided an application in drilling petroleum engineering dynamics.

It is well known that fractional differential equations have recently been proved to be very valuable tools in the modeling of many phenomena in various fields of science and engineering such as mechanics and physics, electrochemistry and engineering, biology, economics, control theory, signal and image processing, aerodynamics, blood flow phenomena, fitting of experimental data, seepage low in porous media and in fluid dynamic traffic model (see, e.g. [18,29,44]). Since the concept of fuzzy fractional differential equation was introduced by Agarwal et al. [5] in 2010, various theoretical results, numerical algorithms and applications have been studied extensively for the fuzzy fractional differential equations by many authors under different conditions. For example, Arshad [1] studied the existence and uniqueness of the solution for a class of fuzzy fractional differential equations and Allahviranloo et al. [2] considered a class of fuzzy Caputo fractional differential equations under the generalized Hukuhara differentiability. Li and Li [32] investigated the stability for fractional order systems based on T–S fuzzy model and Khan et al. [27] proposed a class of max-min improved Euler methods for solving the fuzzy fractional differential equations. Recently, Ahmadian et al. [6] studied a fuzzy fractional kinetic model to calculate the yield and concentration of xylose from the Oil Palm Frond. Some fuzzy fractional functional integral and differential equations have been investigated by Ngo [42,43] and a class of random fuzzy fractional integral equations has been studied by Malinowski [38]. Salahshour et al. [47] presented an analytical solution of the fractional differential equation with uncertainty and provided an application to the basset problem, and Chehlabi and Allahviranloo [10] investigated fuzzy linear fractional differential equations. Very recently, Salahshour et al. [46] studied the properties of Caputo–Fabrizio derivative for interval-valued functions and considered a class of fractional differential equations under this notion, and Long et al. [35] studied a class of fuzzy fractional partial differential equations under Caputo gH-differentiability.

On the other hand, we know that projective dynamical systems have played important roles in modeling of the anomalous phenomena and in the theory of the complex systems during the last two decades. Both wide applications and in-depth theories of the projective dynamical systems have been studied extensively (see, for example, [20–22, 53,54,57,58] and references therein). In particular, in 1994, to consider day-to-day adjustment processes of network flows from one disequilibrium state to another, Friesz et al. [22] proposed a class of globally projected dynamical systems. Later, Xia et al. [53,54] analyzed the global convergence and stability of this systems, and they presented an application to the constrained optimization and the nonlinear complementarity. Very recently, Wu et al. [48–51], Wu et al. [52] extended the globally projected dynamical systems to the global fractional-order projective dynamical systems which capture the desired features of both the variational inequality and the fractional-order dynamical systems within the same framework. They showed some interesting results concerned with the existence, uniqueness of solutions for the global fractional-order projective dynamical systems under mild conditions and also provided an application in adjustment processes of network flows.

However, to our best knowledge, for the fractional-order projective dynamical systems under fuzzy situation, there are very few results in the existing literatures. The motivation of the present work is to make an attempt in this direction.

In this paper, combining the fractional-order projective dynamical systems and differential inclusions for fuzzy mappings (1.1), we consider the following fuzzy fractional differential inclusions with projection operators:

$$\begin{cases} F_{(t,x(t))}\left({}_0^C D_t^q x(t) + g(x(t)) - P_K[g(x(t)) - \rho M(x(t))]\right) > \alpha(x(t)), \\ \text{i.e., } {}_0^C D_t^q x(t) + g(x(t)) - P_K[g(x(t)) - \rho M(x(t))] \in \left(F_{(t,x(t))}\right)_{\alpha(x(t))}, \text{ for } a.e.\ t \in [0,h], \\ x(0) = x_0 \end{cases} \tag{1.2}$$

and

$$\begin{cases} F_{(t,x(t))}\left({}_0^C D_t^q x(t) + g(x(t)) - P_K[g(x(t)) - \rho M(x(t))]\right) \geq \alpha(x(t)), \\ \text{i.e., } {}_0^C D_t^q x(t) + g(x(t)) - P_K[g(x(t)) - \rho M(x(t))] \in \left[F_{(t,x(t))}\right]_{\alpha(x(t))}, \text{ for } a.e.\ t \in [0,h], \\ x(0) = x_0, \end{cases} \tag{1.3}$$

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