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Fuzzy differential equations for universal oscillators ☆

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Abstract

This study considers universal oscillator fuzzy differential equations. The differential inclusion method is applied to this problem by considering the universal oscillator fuzzy differential equations as universal oscillator uncertain dynamical systems. Three types of damped oscillators are considered comprising underdamped, critically damped, and overdamped oscillators. The existence and uniqueness of the solutions to each case are discussed and proofs are given under some certain conditions. For universal oscillator uncertain dynamical systems with an undamped forcing function, the existence and uniqueness of the solutions and big solutions are also verified, while the scopes of the trajectories to the solutions are obtained based on the inclusion relationship between solutions and big solutions. We also prove the existence and uniqueness of solutions to universal oscillator uncertain dynamical systems with a damped forcing function by using other more direct conditions.

Keywords: Differential inclusion; Fuzzy differential equation; Uncertain dynamical system; Universal oscillator

1. Introduction

Fuzzy differential equations (FDEs) are used widely to study mathematical models with uncertainty or subjective information. Buckley et al. [5] studied first order FDEs. Chen et al. [12] and Khastan et al. [19] investigated the boundary value problem for second order FDEs. Fractional and higher order FDEs were discussed in previous studies [2,16]. In the present study, we consider the following universal oscillator FDE:

$$x'' + 2rx' + k^2x = f(t, x, x') \text{ (where } t \in I \subset \mathbf{R},\ r \geq 0,\ k,\ r \in \mathbf{R},\ f : I \times \mathbf{E}_c \times \mathbf{E}_c \to \mathbf{E}_c),$$

with three types of two-point boundary constraints for different relationships between k and r.

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There are three main ways of dealing with FDEs: Zadeh's extension principle (see [6,7,23,24]), H-derivatives and Bede's generalized derivatives (see [2,3,8,12,15,16,19,21,22]), and differential inclusions (see [1,9–11,13,14,17,18, 20]). In previous studies [5,9,11], different basic theories were presented for these three approaches. The H-derivatives approach is used widely to study FDEs but it also has obvious defects. In particular, Diamond and Watson [13] showed that H-derivatives cannot reflect the periodicity, stability, and bifurcation behaviors of FDEs because of the nondecreasing support sets of the fuzzy solutions, although the differential inclusion method can overcome these limitations. For example, the simplest fuzzy two-point boundary value problem comprising: $x'' = (-1) \otimes x$, $x(0) = x(\pi/2) = A$ (where $x : [0, \pi/2] \to \mathbf{E}_c$, $A \in \mathbf{E}_c$ and $[A]^\alpha = [\alpha, 2-\alpha]$, and "$\otimes$" is the operation of product based on Zadeh's extension principle), has no solutions using H-derivatives according to reference [4] and Theorem 4.1 in [12], but it can be solved by the differential inclusion method (see [20]).

Thus, the periodic behavior of semi-linear FDEs and the two-point boundary value problems of FDEs can be successfully solved by using the differential inclusion method (see [9,11,14,20]). In addition, the differential inclusion method is also effective for handling the universal oscillator FDE given above with different two-point boundary constraints. Similar to previous studies [9,11,13,17,20,27], we also treat universal oscillator FDEs as the corresponding universal oscillator uncertain dynamical systems, i.e.,

$$\xi'' + 2r\xi' + k^2\xi \in f(t, \xi, \xi'),$$

where $t \in I \subset \mathbf{R}$, $r \geq 0$, k, $r \in \mathbf{R}$, $f : I \times \mathbf{R} \times \mathbf{R} \to \mathbf{E}_c$, with different two-point boundary constraints. In the uncertain dynamical systems given above, for $\xi \in \mathbf{R}$, $u \in \mathbf{E}_c$, $\xi \in u$ means that $u(\xi) = \mu_u(\xi) > 0$, where μ_u is the membership function of u. The different relationships between r and k can lead to different types of damped uncertain dynamical systems: underdamped, critically damped, and overdamped. These three cases are studied using different Green functions and boundary constraints.

For uncertain dynamical systems, we give definitions of their solutions, as well as proving the existence and uniqueness of their solutions. For uncertain dynamical systems with an undamped forcing function, we provide definitions of their solutions and big solutions, as well as verifying the existence and uniqueness of their solutions and big solutions. The scopes of the trajectories for the solutions are obtained by controlling the big solutions. The more direct conditions for proving the existence and uniqueness of solutions for uncertain dynamical systems with damped forcing function are presented.

The remainder of this paper is organized as follows. Section 2 provides the basic relevant background information for this study. In Section 3, we consider underdamped, critically damped, and overdamped uncertain dynamical systems. For each case, a special type of uncertain dynamical system is investigated. In Section 4, we give our conclusion.

2. Preliminaries

The concepts of fuzzy numbers and relative derivatives for fuzzy number valued functions are explained in this section. In addition, we present some basic properties and theories that are used in this study. Their proofs can be found in previous studies [9–12].

2.1. New parametric representation for fuzzy number valued functions

In this study, the fuzzy space considered is $\mathbf{E}_c$. Before introducing the definition and properties of $\mathbf{E}_c$, we give the following definition and theorem, which are used to obtain the definition of $\mathbf{E}_c$ and to solve the universal oscillator FDEs.

Definition 2.1.1. [13] Let $\mathbf{D}^1$ be the set of upper semicontinuous normal fuzzy sets with compact supports in $\mathbf{R}$ and $\mathbf{E}^1$ are the set of fuzzy convex subsets of $\mathbf{D}^1$.

Theorem 2.1.1. *[Stacking Theorem] [13] Let* $\{A_\alpha \subset \mathbf{R} | \ 0 \leq \alpha \leq 1\}$ *be a class of nonempty compact sets that satisfy:*

(i) $A_\beta \subset A_\alpha$ $(0 \leq \alpha \leq \beta \leq 1)$,

(ii) $A_\alpha = \bigcap_{n=1}^{\infty} A_{\alpha_n}$ *for any nondecreasing sequence* $\{\alpha_n\}$ *in* $[0, 1]$ *that satisfies* $\alpha_n \to \alpha$.

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