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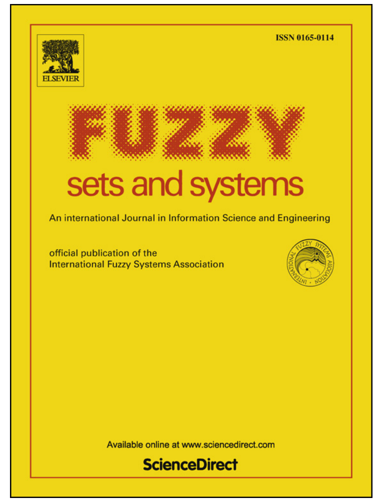

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AU: “On the” is an archaic form in titles. The title would be better as “The fuzzy Poisson equation.”

On the fuzzy Poisson equation

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Abstract

The fuzzy-valued vector function is defined and then divergence, Laplace, and gradient operators are defined for fuzzy-valued vector functions and fuzzy-valued functions. Moreover, a fuzzy Riemann line integral and its fundamental theorem are introduced. To complete our discussion, fuzzy Green’s, fuzzy divergence, and fuzzy Green’s identity theorems are proved. In detail, a fuzzy Poisson equation is considered by discussion of fuzzy maximum and minimum principles. Also, the uniqueness and stability of the solution of a fuzzy Poisson equation are investigated as theorems. Finally, for more illustration, some examples are solved.

Keywords: Fuzzy partial differential equation, Fuzzy Poisson equation, Fuzzy-valued vector function, Fuzzy Green’s theorem, Fuzzy divergence theorem, Fuzzy Green’s identity

1. Introduction

Differential equations with partial derivatives play an important role in modeling many physical phenomena, and in most branches of engineering. On the other hand, the lack of adequate information, uncertainties in the data, measurement errors, etc., result in uncertainties in provided models. These uncertainties in a differential equation can appear in every part of the equation, including the initial and boundary conditions, the coefficients in the equation, and the shape and dimension of the domain. Fuzzy modeling is an effective method for modeling problems with uncertainties in a way that can provide researchers with a more realistic view of the problem.

The fuzzy set theory was first introduced by Zadeh [25]. The concept of the fuzzy derivative was then studied by Chang and Zadeh [20]. Next, Dubois and Prade [10] extended the definition of the fuzzy derivative using Zadeh’s extension principle. One of the first definitions of difference and derivative for set-valued functions (H-difference and H-derivative) was given by Hukuhara [13]. These concepts were then extended by many authors and researchers, such as Puri and Ralescu [19] and Kaleva [15] in fuzzy differential equations. Bede and Gal [4] framed the concepts of weak and strong generalized differentiability. Despite some advantages, these definitions were associated with a number of shortcomings. To alleviate this weakness, Stefanini [22] proposed a generalization of the Hukuhara difference. Then Stefanini and Bede [24] introduced generalized Hukuhara-type differentiability concepts of the interval-valued functions and studied interval differential equations with the differentiability concepts. Stefanini [23] developed a definition of difference called *gH-difference*, which was an extended definition of Hukuhara’s difference. Later, Bede and Stefanini [5] suggested a newer definition. This difference, called *g-difference*, was proposed for fuzzy-valued functions.

The methods of soolving fuzzy ordinary and fuzzy partial differential equations were extended, parallel to the development of fuzzy derivative definitions and concepts. From a historical point of view, fuzzy partial differential equations were first proposed by Buckley and Feuring [7]. Allahviranloo [1] proposed difference methods for solving partial differential equations, Faybishenko [11] proposed a fuzzy partial differential equation model for simulation

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