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A new approach to linear interval differential equations as a first step toward solving fuzzy differential

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Abstract

This study uses a concept of interval differentiability, which was recently introduced, to formulate interval initial value problems involving linear interval differential equations. Differently from the approaches that use the gH-differentiability, this study does not make use of a criterion of choice for switching points in order to obtain solutions for such problems. The method herein presented provides solutions in a simple, straightforward, and computationally tractable way. Moreover, these solutions are intuitive because they coincide with the solutions given by a differential inclusion method. The efficiency and practicality of our approach are illustrated through some examples that have appeared in other articles.

Keywords: Interval-valued functions; Interval differentiability; Interval linear differential equations; Interval initial value problems

1. Introduction

From [19,21,20], it is clear that interval analysis is an important first step in fuzzy interval analysis. Among the topics of fuzzy interval analysis the fuzzy differential equations is one which has a very strong relation with the interval differential equations. One of the motivations for this is that the fuzzy differential equations originally proposed in [24], whose approach is based on the Hukuhara differentiability, have some disadvantages since their solution does not reflect the behavior of the problem initially modeled, see, for example, the radioactivity decay model [13]. In order to provide a model whose solutions describe better the behavior of the problem initially modeled, some articles such as ([14,17]), present different formalizations to work with fuzzy differential equations. In [18] a new formalization

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for fuzzy differential equations is presented using the α-levels of the fuzzy terms contained in the studied equation. In this sense we consider this work as a first step towards solving fuzzy differential equations since the α-levels of a fuzzy number are intervals, and for this reason, we begin with the interval approach.

Differential equation models with interval uncertainties have been studied by many authors, see for example [13,2–4,8,7,9,16,23,25]. These articles use differential inclusion methods or the generalized Hukuhara derivative (gH-derivative, for short).

The behavior of the solutions generated using the concept of the gH-differentiability is directly associated with the concept of switching points. The switching points of a gH-differentiable interval-valued function are the points where the gH-derivatives of such function change from type (i) to type (ii) and vice versa (see [25,5,6]). Using gH-differentiability and selecting in a artificially way some points as the switching points for a solution, it is possible to work with periodic boundary value problems involving linear differential equations with uncertainty (see [16]) and, in particular, it is possible with interval periodic differential equations. On the other hand, since these periodic interval solutions are given through a previous choice of switching points, a wrong choice may generate a solution whose behavior does not represent the problem initially modeled. Thus, the difficulty which arises is to know how to determine the best criterion of choice for the switching points in order to construct a reliable solution. The method developed in this study to obtain interval solutions for interval differential equations avoid this kind of mishap since the method provide interval solutions without having to use a criterion of choice for switching points. Each one of these solutions is obtained by solving only one classical linear differential equation in $\mathbb{R}^2$. Therefore, the solutions of the interval initial value problems involving linear interval differential equations are obtained in a simple, straightforward, and computationally tractable way.

Unlike the interval differential equations methods involving the gH-derivative, to solve interval differential equations via differential inclusion methods allows us to characterize in the interval space the main properties related to ordinary differential equations such as periodicity, stability, bifurcation, among others, in a natural way, ([13], [14], [23], [15]). However, the differential inclusion methods do not deal directly with a concept of derivative of interval-valued functions. That is, the interval differential equations are directly interpreted without having a concept of Interval derivative involved in the process. The method developed in this study uses a concept of interval derivative with which are generated solutions that coincide with the solutions given by a differential inclusion method. Thus our approach allows to interpret the interval differential equations by using directly a interval derivative and also it allows us to generalize in a natural way some properties related to classical differential equations such as those above cited, through such interval derivative.

In short, in this research we study interval initial value problems involving linear interval differential equations by using the concept of φP-differentiability of interval-valued functions, which was recently introduced in [11]. This concept of differentiability is equivalent to gH-differentiability. However, using the φP-differentiable, we obtain solutions for interval initial value problem in a simple, straightforward, and computationally tractable way. Moreover, such solutions coincide with the solutions given by a differential inclusion method.

This research is organized as follows: Section 2 recalls the concepts of limit and of φ-differentiability of generalized interval-valued functions and some results showing that these concepts are equivalent to the concepts of limit and differentiability of vector-valued functions, respectively. In Section 2 is also recalled the concepts of limit and of φP-differentiability of interval-valued functions. The concept of limit of interval-valued functions is obtained by using the concept of limit of generalized interval-valued functions and the φP-differentiability is obtained by using the concept of limit of interval-valued functions. Moreover, it is recalled the result which shows that on particular conditions, the existence of φ-differentiability of a generalized interval-valued function implies the existence of φP-differentiability of an interval-valued function and this result plays a key role in the development of our study. Section 2 is finished recalling a result which states that the φP-differentiability is equivalent to π-differentiability, to gH-differentiability, and to Markov-differentiability.

Section 3 formalizes the generalized interval initial value problems and through this is also formalized the interval initial value problems. Also in Section 3 is presented the method which allows us to obtain solutions of interval initial value differential equation problems by solving a simple initial value problem in $\mathbb{R}^2$. This is one of our main results. Still in the Section 3 is proved that the solutions given by our method coincide with the solutions given by a differential inclusion method, and this is another of our main results. Section 4 presents some numerical examples showing in practice the efficiency of the method herein presented. Finally, in the Section 5 final considerations and future directions are given.

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