Available online at www.sciencedirect.com
ScienceDirect
Fuzzy Sets and Systems ●●● (●●●●) ●●●–●●●
FUZZY sets and systems
www.elsevier.com/locate/fss

Generalized vector complementarity problem in fuzzy environment ☆

Yunru Bai, Stanisław Migórski, Shengda Zeng *

Jagiellonian University in Krakow, Faculty of Mathematics and Computer Science, Chair of Optimization and Control, ul. Lojasiewicza 6, 30-348 Krakow, Poland

Received 24 April 2016; received in revised form 10 June 2017; accepted 27 September 2017

Abstract

This paper is devoted to introduce and study a new class of generalized vector complementarity problems ((GVCP), for short) and generalized vector variational inequalities ((GVVI), for short) in fuzzy environment. Under suitable conditions, we prove the equivalence between (GVCP) and (GVVI) in Banach spaces. Then, without any monotonicity assumption, we apply KKM-technique to establish an existence theorem of solutions for (GVVI). Finally, we show that the solution set of (GVCP) is nonempty and closed.

Keywords: Generalized vector complementarity problem; KKM theorem; Generalized vector variational inequality; Fuzzy mapping

1. Introduction

Since the concept of fuzzy sets was introduced initially by Zadeh in 1965, a wide range of new theories dealing with uncertainty and imprecision have been discussed. Over the years, fuzzy set theory have attracted increasing attention mainly due to its many applications in information sciences and control theory, such as, pattern recognition, artificial intelligence, optimization, decision theory, computer science, operations research and so forth (for more details, one can refer to [1,25,26,32,33]).

The theory of variational inequalities has emerged as one of the most promising branches of pure and applied mathematics. It provides us with a convenient mathematical apparatus for studying a large number of problems arising in diverse fields, for instance, contact mechanics in elasticity, optimal control, inverse problems, financial mathematics and others (see [12,22,27] and references therein). Equally significant is the area of mathematical programming known as the complementarity theory, which was introduced by Lemke [20] and Cottle–Danzig [10], has been studied and applied in nonlinear analysis for more than 40 years, for example, fluid flow through porous media, economics and transportation equilibria, lubrication problem and so on. For more details, we refer the reader to references [5,14–17,19].

☆ Project supported by the National Science Center of Poland under Maestro Project No. UMO-2012/06/A/ST1/00262.
* Corresponding author.
E-mail addresses: yunrubai@163.com (Y. Bai), stanislaw.migorski@uj.edu.pl (S. Migórski), zengshengda@163.com (S. Zeng).

https://doi.org/10.1016/j.fss.2017.09.010
0165-0114/© 2017 Published by Elsevier B.V.

Motivated by the above mentioned, in 1989, Chang–Zhu [2] firstly introduced the concept of variational inequalities for fuzzy mappings in abstract spaces and studied the existence problem for solutions of some class of variational inequalities for fuzzy mappings. Then, Chang–Lee–Lee [3] considered a class of vector quasi-variational inequalities for fuzzy mappings and obtained the fuzzy extensions of the well known Walras theorem, which is an important one in the mathematical economics. Furthermore, Chang–Cho–Lee–Lee [4] introduced the concept of fixed degrees for fuzzy mappings in probabilistic metric spaces and established several fixed point theorems for fuzzy mappings. Recently, several types of variational inequalities and complementarity problems for fuzzy mappings were introduced and studied by Chang–Salahuddin [6], Chang–Salahuddin–Ahmad–Wang [7], Patriche [24], Tang–Zhao–Wan–He [29], and Kılıçman–Ahmad–Rahaman [18].

The purpose of this paper is to consider a new class of generalized vector complementarity problems ((GVCP), for short) and generalized vector variational inequalities ((GVVI), for short) in fuzzy environment. We establish an equivalence between (GVCP) and (GVCP), then derive some existence results for our problems. The results presented in the paper extend and improve the corresponding results in [13–15,17–19,21].

Let X, Y be real Banach spaces and K be a nonempty closed, convex subset of X. Given a fuzzy mapping $T\colon K \to \mathfrak{F}(L(X,Y))$, a function $\alpha\colon K \to (0,1]$, and a set-valued mapping $C\colon K \to P(Y)$ with nonempty pointed, convex cone values, in this paper, we consider the following generalized vector complementarity problem in fuzzy environment.

Problem $\mathcal{P}$. Find $x \in K$ and $t \in (T_x)_{\alpha(x)}$ such that

$$\begin{cases} \langle N(At,Bt), g(x)\rangle + F(g(x),g(x)) = 0, \\ \langle N(At,Bt), h(y)\rangle + F(h(y),g(x)) \in C(x), \quad \text{for all } y \in K, \end{cases}$$

where $(T_x)_{\alpha(x)}$ is the $\alpha(x)$-cut set of fuzzy set T_x defined by

$$(T_x)_{\alpha(x)} := \{t \in L(X,Y) : T_x(t) \ge \alpha(x)\}.$$

The second problem which we study, is called generalized vector variational inequality defined as follows.

Problem $\mathcal{Q}$. Find $x \in K$ and $t \in (T_x)_{\alpha(x)}$ such that

$$\langle N(At,Bt), h(y)-g(x)\rangle + F(h(y),g(x)) - F(g(x),g(x)) \in C(x), \quad \text{for all } y \in K.$$

The operators N, A, B, h, g, F are given, which will be specified in Section 3

$$\begin{aligned} &N\colon L(X,Y) \times L(X,Y) \to L(X,Y), \\ &A\colon L(X,Y) \to L(X,Y), \\ &B\colon L(X,Y) \to L(X,Y), \\ &h\colon K \to K, \\ &g\colon K \to K, \\ &F\colon K \times K \to Y, \end{aligned}$$

where $L(X,Y)$ is the space of all bounded linear operators from X to Y.

We would like to point out that Problem $\mathcal{P}$ and Problem $\mathcal{Q}$ include many kinds of generalized vector complementarity problems and generalized variational inequalities as their special cases.

(i). If $N(At,Bt) = t$ for all $t \in L(X,Y)$, $h(x) = g(x)$ for all $x \in K$, and $F\colon K \to Y$, then Problem $\mathcal{P}$ is equivalent to
Problem $\mathcal{P}_1$. Find $x \in K$ and $t \in (T_x)_{\alpha(x)}$ such that

$$\begin{cases} \langle t, g(x)\rangle + H(x) = 0, \\ \langle t, g(y)\rangle + H(y) \in C(x), \quad \text{for all } y \in K, \end{cases}$$

where $H(x) = F(g(x))$ for all $x \in K$.
In the meantime, Problem $\mathcal{Q}$ reduces to
Problem $\mathcal{Q}_1$. Find $x \in K$ and $t \in (T_x)_{\alpha(x)}$ such that

Please cite this article in press as: Y. Bai et al., Generalized vector complementarity problem in fuzzy environment, Fuzzy Sets Syst. (2017), https://doi.org/10.1016/j.fss.2017.09.010