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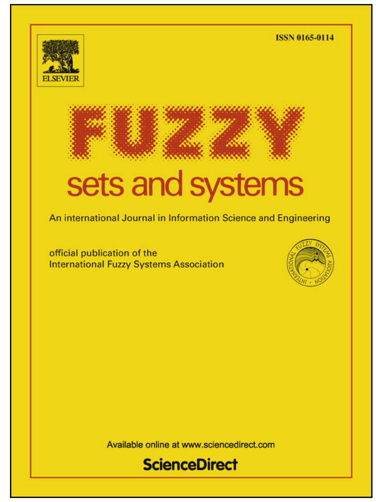

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Cauchy-like functional equations for uninorms continuous in $(0,1)^2$

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Abstract

Commutativity is an important property in two-step information merging procedure. It is shown that the result obtained from the procedure should not depend on the order in which signal steps are performed. In the case of a bisymmetric aggregation operator with the neutral element, Saminger et al. have provided a full characterization of commutative n-ary operator by means of unary distributive functions. Further, characterizations of these unary distributive functions can be viewed as resolving a kind of the Cauchy-like equations $f(x \oplus y) = f(x) \oplus f(y)$, where $f\colon [0,1] \to [0,1]$ is a monotone function, $\oplus$ is a bisymmetric aggregation operator with the neutral element. In this paper, we are still devoted to investigating and fully characterizing the Cauchy-like equation $f(U(x,y)) = U(f(x), f(y))$, where $f\colon [0,1] \to [0,1]$ is an unknown function but not necessarily monotone, U is a uninorm continuous in $(0,1)^2$. These results show the key technology is how to find a transformation from this equation into several known cases. Moreover, this equation has completely different and non-monotone solutions in comparison with the obtained results.

Keywords: Fuzzy connectives, Commutativity, Distributivity, Cauchy-like functional equations, T-norms, Uninorms

1. Introduction

Commutativity is widely used in various fields, such as information fusion and multifactorial evaluation (see [6], [10], [21], [22], [25], [32]). It reveals that the terminal results at last have nothing to do with the order of the steps. With this property, then in practice the two steps can be exchanged if there is no reason to perform either of the steps first. For example, if we want to judge the performance of the third grade in an elementary school, we can first evaluate different courses of each student in this grade and then give the result for each one, and finally merge the data of all students and obtain the final evaluation. On the other hand, we also can analyse and estimate each course of all students, then put them together and calculate the determined result. In many situations, one always hopes that two procedures yield the same result. This property is said to be *commutative*.

Recently, the commutative aggregation operators caught more and more attentions. For instance, they are used to preserve the transitivity when aggregating preference matrices or fuzzy relations (see [8], [9], [34]) or some form of additivity when aggregating set functions (see [12], [24]). Specially, Saminger, Mesiar and Dubois [35] investigated the commutativity of aggregation operators with bisymmetry. In the case of a bisymmetric aggregation operator with the neutral element, they have provided a full characterization of commutative n-ary operator by means of unary distributive functions. Further, it has been shown that characterizations of these unary distributive functions can be viewed as resolving a kind of the generalized Cauchy equations

$$f(x \oplus y) = f(x) \oplus f(y), \tag{1}$$

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