Accepted Manuscript

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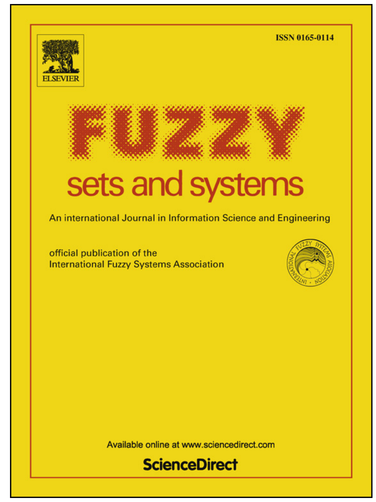

PII: S0165-0114(17)30321-4
DOI: http://dx.doi.org/10.1016/j.fss.2017.08.007
Reference: FSS 7287

To appear in: *Fuzzy Sets and Systems*

Received date: 21 February 2017
Revised date: 2 June 2017
Accepted date: 16 August 2017

Please cite this article in press as: R. Mesiar et al., Aggregation functions and capacities, *Fuzzy Sets Syst.* (2017), http://dx.doi.org/10.1016/j.fss.2017.08.007

Aggregation Functions and Capacities

Radko Mesiar*† Surajit Borkotokey‡ LeSheng Jin§ Martin Kalina¶

Abstract

Inspired by earlier results of Bayes and Benvenuti et al.[3], we introduce and study some construction methods and transformations for capacities. First we introduce constructions of capacities based on n-ary aggregation functions and implications from fuzzy set theory, parameterized by vectors $\mathbf{x} \in [0,1]^n$. Later, implication based transforms of capacities, parameterized by subsets $B \subseteq \{1,2,...,n\}$, are given, exemplified and discussed.

Keywords: Aggregation function; Capacity; Conditional Probability; implications.

1 Introduction

Formally, any n-ary aggregation function $A : [0,1]^n \to [0,1]$, see [2, 9], can be seen as a monotone extension of a capacity $m_{(A)}$ acting on the space $\mathcal{N} = \{1,2,...,n\}$, where $m_{(A)}(E) = A(\mathbf{1}_E)$, and $\mathbf{1}_E : \mathcal{N} \to \{0,1\}$ is the characteristic function of the set E. Then, $m_{(A)}(E) = \dfrac{A(\mathbf{x} \wedge \mathbf{1}_E)}{A(\mathbf{x})}$ where $\mathbf{x}$ is the top vector of $[0,1]^n$ i.e., $\mathbf{x} = \mathbf{1} = (1,1,...,1)$. As observed by Benvenuti et al. [3], any n-ary aggregation function A is related to a family $(m_{A,c})_{c\in[0,1]}$ of capacities on $\mathcal{N}$, where $A(\mathbf{c} \wedge \mathbf{1}_E) = A(\mathbf{c})\, m_{A,c}(E)$ with $\mathbf{c} = (c,c,c,...,c)$. Clearly, if $A(\mathbf{c}) \neq 0$, then $m_{A,c}$ is unique,

$$m_{A,c}(E) = \frac{A(\mathbf{c} \wedge \mathbf{1}_E)}{A(\mathbf{c})} \tag{1}$$

Note that if $A(\mathbf{c}) = 0$, $m_{A,c}$ can be chosen arbitrarily. Similar ideas, but based on overlap functions can be found in [12]. Recall that also in classical probability theory we have methods for deriving parametric families of probability measures. Consider, e.g., the Bayesian conditional probabilities. The aim of this paper is to have a closer look at the connection between aggregation functions

*Faculty of Civil Engineering, Slovak University of Technology, Slovakia, Email: radko.mesiar@stuba.sk
†Faculty of Science, Palacký University, Olomouc, 12,77146 Olomouc, Czechia
‡Department of Mathematics, Dibrugarh University, India, Email: surajitbor@yahoo.com
§Business School, Nanjing Normal University, Nanjing, China, Email: jls1980@163.com
¶Faculty of Civil Engineering, Slovak University of Technology, Slovakia, Email:kalina@math.sk