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Level-based fuzzy generalized quantification

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Abstract

We propose a new type of generalized quantification using a recently developed representation of fuzziness by levels. This proposal integrates the existent models allowing the extension of crisp quantification to the fuzzy case keeping all the algebraic Boolean properties. Besides, the model covers new types of quantification not covered by the existent approaches. The advantages of our proposal are clear from the computational and practical perspective, since it allows a parallelization of the algorithm by taking into account a fixed number of levels.

Some authors have proposed several properties that a good evaluation method should satisfy. We also prove that our proposal fulfills all the desired properties to be considered a good evaluation method.

Keywords: Fuzzy quantification; Generalized quantifiers; Quantified sentences; Representation by levels

1. Introduction

Quantified sentences are a very powerful notion for modeling statements in Natural Language (NL). Particularly, they have been used in the framework of Computing with Words and Perceptions [24]. A quantified sentence is a linguistic expression modeling a quantification statement by employing the so-called quantifiers. Examples of quantifiers are *all, a few, many,* etc. Depending on the type of quantifier we can build different expressions, arising different kinds of quantification. The process of calculating the accomplishment degree of a quantified sentence is usually performed by using an evaluation method. Sentence evaluation methods have been widely applied in many fields. A good overview of the areas of application of evaluations methods can be found in [4]. To name some of them, information retrieval and linguistic summarization are some of the more active ones.

Fuzzy quantified sentences arise when fuzzy quantifiers are used in the sentence. They offer a more realistic description of the sentence, allowing a relaxation when describing the quantities, having for instance “around a half” instead of “a half”. Some approaches for evaluating the accomplishment degree of fuzzy quantified sentences have been developed in the past years. We can distinguish between two different approaches: those following Zadeh’s framework [22,23] and those using the Theory of Generalized Quantifiers (TGQ) [3,15]. In fact, in [4] it is discussed

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how the quantified sentences described following the Zadeh's framework can be considered as particular cases of sentences expressed using the modeling of TGQs.

In both frameworks several properties have been proposed to be fulfilled by the quantification methods in order to obtain intuitive results from the human point of view. Depending on the followed framework we can find a different set of properties, but the majority of them correspond to the same idea. We refer the reader to [4] for a complete view of the different properties and the set of quantifiers satisfying them. Some of these properties are those of Boolean algebras or can be proved to be dependent on them and their fulfillment depends not only on the evaluation method, but on the operations employed on fuzzy sets. Fuzzy Set Theory (FST) has been extensively developed and some problems have arisen when some desirable properties were not fulfilled by an FST. For instance, the expected accomplishment degree of the following sentences should be 1:

- *all* A are $A \wedge A$
- *all* X are $A \vee \neg A$
- *no* X is $A \wedge \neg A$

where $\wedge$, $\vee$ and $\neg$ are calculated by means of an FST, the quantifier all is defined on [0,1] as $all(\alpha) = 1$ iff $\alpha = 1$, and 0 otherwise, and the quantifier no can be defined as the negation of the quantifier $exists$ as $no(\alpha) = 1$ iff $\alpha = 0$ and 0 otherwise. But the accomplishment degree of 1 for an FST is satisfied if and only if the following three conditions hold simultaneously:

- $card(A \wedge A)/card(A) = 1$, which implies $A \wedge A = A$ (idempotency),
- $card(A \vee \neg A)/card(X) = 1$, which implies $X = A \vee \neg A$ (excluded middle),
- $card(A \wedge \neg A)/card(X) = 0$, which implies $A \wedge \neg A = \emptyset$ (non-contradiction).

However, this is not possible since Dubois and Prade showed in [6,7] that no standard FST can satisfy idempotency (as well as mutual distributivity) together with the laws of excluded middle and non-contradiction, although some of them can be fulfilled with particular choices of t-norms (like in the case of Lukasiewicz conjunction and the property $A \otimes \neg A = \emptyset$). As a consequence, when using fuzzy sets, there are intuitive properties that cannot be satisfied by the evaluation of fuzzy quantified sentences.

To solve all these problems we develop a proposal that

1. guarantees the fulfillment of Boolean algebra properties and therefore we obtain intuitive results when evaluating sentences like the ones introduced in the example above,
2. integrates previous proposals following the Zadeh's framework and the Theory of Generalized Quantifiers, and
3. satisfies the desired properties for being a good evaluation method.

Our proposal uses a recently developed model for representing fuzziness called Representation by Levels (RL). The RL theory model allows elements to satisfy a concept and its negation at the same time to some degree but keeping all Boolean properties [18,21]. In fact, the RL theory has the structure of a Boolean Algebra (see section 2.2). Fuzzy sets can also be represented in this theory but the results obtained when performing operations are different to those obtained when employing a FST. In particular, the result cannot be a fuzzy set. Moreover, our model is easily parallelizable by taking into account a fixed number of levels. This offers a clear advantage from the computational and practical perspective, feature that is very important in fields like fuzzy expert systems, natural language processing, fuzzy temporal knowledge representation and reasoning, etc.

The structure of the paper is the following: first we present the preliminary concepts necessary for the comprehension of the paper, including a brief overview of Zadeh's and TGQs frameworks and a description of the Representation by Levels (RL) theory. The core of this work is in Section 3 where our proposal for evaluating fuzzy quantified sentences using the RL theory is presented. Later in Section 4 we study the properties satisfied by our proposal. We finish with the conclusions and future research on this topic.

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