Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems ••• (••••) •••–•••

FUZZY sets and systems

www.elsevier.com/locate/fss

Qualitative conditioning in an interval-based possibilistic setting

Salem Benferhat [a,b], Vladik Kreinovich [c], Amélie Levray [a,b], Karim Tabia [a,b,*]

[a] *Univ. Lille Nord de France, F-59000 Lille, France*
[b] *UArtois, CRIL–CNRS UMR 8188, F-62300 Lens, France*
[c] *Department of Computer Science, University of Texas at El Paso, 500 W. University El Paso, TX 79968, USA*

Received 14 May 2016; received in revised form 2 November 2017; accepted 7 December 2017

Abstract

Possibility theory and possibilistic logic are well-known uncertainty frameworks particularly suited for representing and reasoning with uncertain, partial and qualitative information. Belief update plays a crucial role when updating beliefs and uncertain pieces of information in the light of new evidence. This paper deals with conditioning uncertain information in a qualitative interval-valued possibilistic setting. The first important contribution concerns a set of three natural postulates for conditioning interval-based possibility distributions. We show that any interval-based conditioning satisfying these three postulates is necessarily based on the set of compatible standard possibility distributions. The second contribution consists in a proposal of efficient procedures to compute the lower and upper endpoints of the conditional interval-based possibility distribution while the third important contribution provides a syntactic counterpart of conditioning interval-based possibility distributions in case where these latter are compactly encoded in the form of possibilistic knowledge bases.

Keywords: Interval-based possibilistic logic; Conditioning; Possibility theory

1. Introduction

Many problems and applications need efficient formalisms for encoding and reasoning with uncertain, partial information or knowledge. Possibility theory and possibilistic logic [1–5] are uncertainty frameworks particularly suited for representing and reasoning with uncertain, incomplete, prioritized and qualitative information. In the literature, many extensions have been proposed for possibilistic logic to deal for instance with imprecise certainty degrees [6,7], symbolic certainty weights [8,9], multi-agent beliefs [10], temporal and uncertain information [11], uncertain conditional events [12–14], generalized possibilistic logic [1,4,15], justified beliefs [16], etc.

Interval-based uncertainty representations extend the underlying uncertainty settings in order to encode uncertainty by means of intervals of possible degrees instead of single values. Such extensions allow more flexible representations

* Corresponding author.
E-mail addresses: benferhat@cril.univ-artois.fr (S. Benferhat), vladik@utep.edu (V. Kreinovich), levray@cril.univ-artois.fr (A. Levray), tabia@cril.univ-artois.fr (K. Tabia).

https://doi.org/10.1016/j.fss.2017.12.007
0165-0114/

especially to deal with poor information, imprecise or ill-known beliefs, confidence intervals and multi-source information [17,18]. Such representations are very widely used in some applications such as sensitivity analysis. In this paper, we are interested in interval-based possibilistic logic [6] which extends the standard possibilistic logic setting to allow intervals of possible degrees instead of single values attached to the formulas of the knowledge base.

Conditioning is an important task for updating the current uncertain information when a new sure piece of information is received. A conditioning operator is designed to satisfy some desirable properties such as giving priority to the new information and ensuring minimal change while transforming an initial distribution into a conditional one. Conditioning in standard (single-valued) possibility theory has been addressed in many works [19–27]. There are two major definitions of a possibility theory: min-based (or qualitative) possibility theory and product-based (or quantitative) possibility theory. At the semantic level, these two theories share the same definitions, including the concepts of possibility distributions, necessity measures, possibility measures and the definition of normalization conditions. However, they differ in the way they define possibilistic conditioning. Indeed, in possibility theory, there are two main definitions of possibilistic conditioning. The first one is called min-based conditioning [19,28] (or qualitative-based conditioning) which is appropriate in situations where only the ordering between events is important. In this case, the unit interval [0, 1] is viewed as an ordinal scale where only the minimum and the maximum operations are used for propagating uncertainty degrees. The second definition of conditioning is called product-based conditioning (or quantitative-based conditioning) where the unit interval is used in a general sense. In this case, the product operation can also be used in the propagation of uncertainty degrees. In the context of interval-based possibility theory [6,7,29], an extension of a conditioning operator is proposed for the interval-based setting. This is only done for the *product-based conditioning*. This extension is based on conditioning compatible possibility distributions and a syntactic counterpart for conditioning possibilistic logic bases is also proposed. In [12,13], the authors dealt with some issues regarding inference (propagating possibility and necessity bounds) and independence where the beliefs are encoded using the concept of uncertain conditionals in a possibilistic setting.

This paper is primarily oriented to the study of min-based conditioning in an interval-based possibilistic setting and contains three major contributions:

- The first contribution (Section 4, Theorem 1) deals with conditioning in an interval-based possibility theory setting. We first propose three natural postulates for an interval-based conditioning. We show that any interval-based conditioning satisfying these postulates is necessarily based on applying min-based conditioning on each compatible standard possibility distribution.
- The second contribution (Section 4) consists in providing the exact lower and upper endpoints of min-based conditioning an interval-based distribution and a proposal of efficient procedures to compute the lower and upper endpoints of the conditional interval-based possibility distribution.
- The third contribution (Section 5) concerns syntactic computations of conditioning where interval-based possibility distributions are compactly represented by interval-based knowledge bases. We show that interval-based conditioning has the same computational complexity as the standard min-based conditioning.

Before presenting these contributions, let us first provide a brief refresher on possibility theory and possibilistic logic.

2. Brief reminder on possibility theory and possibilistic logics

Possibility theory [30,31] is a well-known alternative uncertainty theory. This framework was coined by L. Zadeh [31] and it is developed by several researchers (e.g. Dubois and Prade [32], Yager [33] and Borgelt and Kruse [34]). Possibility theory is based on a pair of dual measures allowing to evaluate the knowledge/ignorance relative to the event in hand. Among the main concepts of this framework are the ones of possibility distribution and possibilistic knowledge base.

2.1. Possibility distributions

A possibility distribution, denoted π, is a mapping that attaches to every state ω of the world Ω (the universe of discourse or the set of states of the world) a degree in the unit interval [0, 1] expressing a partial knowledge over the world. The degree $\pi(\omega)$ associated with a state ω represents the degree of compatibility (or consistency) of the

Please cite this article in press as: S. Benferhat et al., Qualitative conditioning in an interval-based possibilistic setting, Fuzzy Sets Syst. (2017), https://doi.org/10.1016/j.fss.2017.12.007