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The cg-position value for games on fuzzy communication structures

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Abstract

A cooperative game for a set of agents establishes a fair allocation of the profit obtained for their cooperation. The best known of these allocations is the Shapley value. A communication structure defines the feasible bilateral communication relationships among the agents in a cooperative situation. Some solutions incorporating this information have been defined from the Shapley value: the Myerson value, the position value, etc. Later fuzzy communication structures were introduced. In a fuzzy communication structure the membership of the players and the relations among them are leveled. Several ways of defining the Myerson value for games on fuzzy communication structure were proposed, one of them is the Choquet by graphs (cg) version. Now we study in this work the cg-position value and its calculation. The cg-position value is defined as a solution for games with fuzzy communication structure which considers the bilateral communications as players. So, the Shapley value is applied for a new game (the link game) over the fuzzy sets of links in the fuzzy communication structure and the profit obtained for each link is allocated between both players in the link. As we see in our examples and results the cg-position value is more concerned with the graphical position of the players and their communications than the other cg-values. In this paper we also introduce a procedure to compute exactly the position value, avoiding to calculate the characteristic function of the link game for all coalitions. This procedure is used to determine the cg-position value. Finally we compare the new value with other cg-values in an applied example about the power of the groups in the European Parliament.

Keywords: Game theory; Fuzzy graphs; Position value; Harsany's dividends; Power indices; European Parliament

1. Introduction

A cooperative game with transferable utility over a finite set of players is defined as a function establishing the worth of each coalition (subset of players). The outcome of a game is a payoff vector, namely it is a vector in which each component represents the payment for each player because of their cooperation possibilities. The Shapley value [1] is the best known of these outcomes. The payoffs of the players in a game for the Shapley value are the expected worths of their marginal contributions to the coalitions containing them, i.e. the differences between the worth of the coalition and the worth of this coalition without each of them (see formula (1) in Section 2). The usual payoff vectors,

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also the Shapley value, suppose that all the communications are feasible and then all the cooperation possibilities. Moreover, they use the worths of all the coalitions in their formulations. Therefore, the payoff vector takes into account how the players cooperate among them and how the coalitions are formed. But usually this fact is not realistic and not all the coalitions are feasible. Several different models of games with partial cooperation have been studied: Aumann and Dreze [2], Owen [3], Myerson [4], etc.

Myerson [4] considered that the communication among the players is not always complete. He described the communication situation by a graph where the vertices are the players and the links are the feasible bilateral communications among them. This graph is named the communication structure of the game. Hence we will use both, graph or communication structure, alike. A communication value assigns a payoff vector to each game with a specific communication structure. The Myerson model supposes that the feasible coalitions are the connected sets in the graph and so only the worths of these coalitions should be used to elaborate outcomes. There exist several communication values defined from the Shapley value: the Myerson value [4], the position value [5], the average tree value [6], etc. The position value focuses the allocation of the profits on the links. A new cooperative game over the communications is defined by calculating the worth of a set of links as the sum of worths of the connected components in the subgraph generated by them. The Shapley value of this new game is a payoff vector for the links and the payoff of each link is allocated between both of he players in the link. Borm et al. [7] gave a characterization of the position value only for communication structures without cycles (trees). Later, Slikker [8] got a characterization for all the situations. Other characterizations or similar solutions can be found in van den Nouweland and Slikker [9], Ghintran et al. [10] and Ghintran [11]. The position value is directly related to the situation of the players in the graph as we can see in [7] where the authors proved that if the game only depends on the cardinality of the coalitions then the payoff of a player is proportional to his degree.

The problem of computing the Shapley value is NP-complete although it is P-complete for several particular cases, for instance the family of weighted majority games (see [12]). The Myerson value and the Shapley value were provided with several exact algorithms (for instance [13–15] and [16]). Particularly these algorithms were used to analyze the power in the European Union in [17] and [18]. References about computational aspects of cooperative game theory can be found in [19], but there is no analysis about algorithms to determine the position value.

Aubin [20] supposed uncertainty about the membership of the players in the coalitions studying games with fuzzy coalitions. To calculate the worth of a fuzzy coalition in a game it is necessary to consider a specific partition by levels of this fuzzy set. One of these partitions was defined by Tsurumi et al. [21] using the Choquet integral. Following this way, the uncertainty about the existence of the communications among the players can be extended. Recently, Jiménez-Losada et al. [22] introduced fuzzy graphs to analyze communication among players. Fuzzy graphs allow leveling the links between being feasible or not, and they also allow considering membership levels for the players. The idea of partition by levels was extended to fuzzy communication structures in [23], proposing different extensions of the Myerson value for fuzzy situations. In one of them, the Choquet by graphs (cg) option, players look for the biggest communication structure at the same level at each moment. Gallego et al. [24] studied the cg-Banzhaf value for fuzzy communication structures and the complexity of its calculation. In these models the Choquet integral determines the worth of a coalition with fuzzy links by intervals of levels of communication. The analysis of fuzzy communication structures can also allow to study games on communication networks with infinite range scaling by a sigmoid curve (the logistic function, for instance). This tool is usual in fuzzy networks as the reader can see in [25].

The main goal of this paper is to study the position value for games with fuzzy communication structure in the Choquet by graphs version. This paper is a logical continuation of our previous works about the Myerson value. We define the new solution using the Choquet integral, we get axioms for the cg-position value and we are also concerned with the computational aspects of the value. The cg-position value is also related with the fuzzy situation of the players, particularly with the fuzzy degree, as we will see later. The solution and the proposed algorithm are showed in an applied example, and it is compared to the other similar solutions. The organization of the paper is the following. Section 2 presents in short the background about cooperative games and communication structures which allows the reader to follow the paper. Section 3 is dedicated to games with fuzzy communication structure and the cg-partition. We define also the cg-position value in this section. We obtain in section 4 an axiomatization of the value. Section 5 analyzes the computation of the cg-position value and the time complexity of the algorithm. Finally in Section 6 we apply our solution to determine the power of the political groups in the European Parliament.

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