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Short Communication

A note on "A group decision making model based on a generalized ordered weighted geometric average operator with interval preference matrices"

Zhou-Jing Wang

School of Information, Zhejiang University of Finance & Economics, Hangzhou, Zhejiang 310018, China

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Abstract

A recent paper by Liu, Zhang and Zhang (2014) [6] introduced a consistency index of an interval multiplicative reciprocal matrix (IMRM). In the context of group decision making, each decision maker supplies an IMRM to describe its preferences. The consistency index of each IMRM is used to rank individual IMRMs. This ordering is then employed in the aggregation process which is based on an ordered weighted geometric average operator. Furthermore, these authors devised an approach to determine importance weights of individual IMRMs. This note shows that such a consistency index highly depends on the numbering of compared objects, and the determination method of importance weights is questionable. A new consistency index is defined and used to rank individual IMRMs. A novel method is developed to obtain importance weights of individual IMRMs, and some properties are provided for the aggregation operator and the aggregated group IMRM.

Keywords: Group decision making; Interval multiplicative reciprocal matrix; Consistency index; Induced ordered weighted geometric averaging operator

1. Introduction

Aggregation of individual pairwise comparison matrices is an important step in solving group decision making (GDM) problems. In this step, group judgments are often obtained from individual opinions by using an aggregation operator. A recent paper by Liu et al. [6] put forward a generalized ordered weighted geometric average operator named the consistency-induced ordered weighted geometric averaging (CIOWGA) operator for aggregating individual interval multiplicative reciprocal matrices (IMRMs) into a collective IMRM. These authors defined the arithmetic weighted average of consistency ratios of two multiplicative reciprocal matrices (MRMs) constructed from an IMRM as a consistency index of the IMRM. This index is used to rank individual IMRMs in terms of importance and served as an order-inducing index for using the CIOWGA operator to aggregate individual preferences. Furthermore, Liu

E-mail address: wangzj@xmu.edu.cn.

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et al. [6] proposed an approach to determine importance weights of individual IMRMs for the CIOWGA operator, and developed a CIOWGA operator based aggregation method to fuse individual IMRMs. However, the proposed consistency index is sensitive to the numbering of compared objects, and the proposed determination method of importance weights is questionable. The aim of this note is to reveal and correct flaws in the results by Liu et al. [6].

The remainder of this note is organized as follows. Section 2 gives preliminaries on notions and terminologies of MRMs and IMRMs, and furnishes some results in [6]. Flaws in the existing results are pointed out and illustrated in Section 3. Section 4 introduces a new consistency measure of IMRMs and a new method of determining importance weights to correct the flaws. Finally, Section 5 provides main conclusions.

2. Preliminaries

This section reviews basic concepts related to MRMs and IMRMs, and gives some results by Liu et al. [6].

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of n objects, if a positive pairwise comparison matrix $A = (a_{ij})_{n\times n}$ on X satisfies

$$a_{ii} = 1, \qquad a_{ij}a_{ji} = 1, \quad i, j = 1, 2, \ldots, n \tag{2.1}$$

then A is called a MRM.

Saaty [7] proposed the following consistency index (CI) to measure inconsistency of a MRM.

$$CI_A = \frac{\lambda^A_{\max} - n}{n-1} \tag{2.2}$$

where $\lambda^A_{\max}$ is the maximum eigenvalue of the MRM A.

It is clear that the higher the value of CI_A is, the more inconsistent the MRM A. To check acceptable consistency of MRMs, Saaty [7] also developed a consistency ratio (CR) as

$$CR_A = \frac{CI_A}{RI(n)} \tag{2.3}$$

where $RI(n)$ is Saaty's random index determined by the average CI of randomly generated MRMs of order n. If $CR_A \le 0.1$, then the MRM A is said to have acceptable consistency; otherwise it is unacceptable.

Saaty and Vargas [8] introduced a notion of IMRMs for dealing with pairwise comparisons with uncertainty. An IMRM $\bar{A}$ on X is represented by an interval-valued matrix $\bar{A} = (\bar{a}_{ij})_{n\times n}$ satisfying

$$\bar{a}_{ij} = [a^-_{ij}, a^+_{ij}], \qquad a^-_{ij} \le a^+_{ij}, \qquad a^-_{ij}a^+_{ji} = 1, \qquad a^-_{ii} = a^+_{ii} = 1, \quad i, j = 1, 2, \ldots, n. \tag{2.4}$$

Induced ordered weighted operators are often used to aggregate individual judgments into a group opinion [1–3, 6,10]. To obtain an importance ranking of individual IMRMs, Liu et al. [6] defined a consistency index of an IMRM $\bar{A} = (\bar{a}_{ij})_{n\times n} = ([a^-_{ij}, a^+_{ij}])_{n\times n}$ (see Eq. (20) on page 10 in [6]). This consistency index can be rewritten by using the notation in this note as

$$CR_{Liu}(\bar{A}) = \frac{CR_{C_{\bar{A}}} + CR_{D_{\bar{A}}}}{2} \tag{2.5}$$

where $C_{\bar{A}} = (c_{ij})_{n\times n}$ and $D_{\bar{A}} = (d_{ij})_{n\times n}$ are two MRMs defined by the following formula (see Eq. (5) on page 4 in [6]).

$$c_{ij} = \begin{cases} a^+_{ij} & i < j \\ 1 & i = j \\ a^-_{ij} & i > j \end{cases}, \qquad d_{ij} = \begin{cases} a^-_{ij} & i < j \\ 1 & i = j \\ a^+_{ij} & i > j \end{cases} \tag{2.6}$$

It can be observed that the consistency index (2.5) is the arithmetic weighted average of the CRs of the two MRMs $C_{\bar{A}}$ and $D_{\bar{A}}$ generated from the upper or lower bounds of the interval judgments in the IMRM $\bar{A}$.

Based on (2.5), Liu et al. [6] developed a CIOWGA operator based method to aggregate m individual IMRMs $\bar{A}_k = (\bar{a}_{ij(k)})_{n\times n} = ([a^-_{ij(k)}, a^+_{ij(k)}])_{n\times n}$ $(k = 1, 2, \ldots, m)$ into a collective IMRM $\bar{A}_c = (\bar{a}_{ij(c)})_{n\times n} = ([a^-_{ij(c)}, a^+_{ij(c)}])_{n\times n}$. Let σ be a consistency-based permutation of $\{1, 2, \ldots, m\}$ such that $CR_{Liu}(\bar{A}_{\sigma(k)}) \le CR_{Liu}(\bar{A}_{\sigma(k+1)})$ for $k = 1, 2, \ldots, m-1$, and $V = (v_1, v_2, \ldots, v_m)^T$ be an importance weight vector satisfying $v_1 \ge v_2 \ge \cdots \ge v_m$, $\sum_{k=1}^m v_k = 1$

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