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Global exponential convergence of fuzzy complex-valued neural networks with time-varying delays and impulsive effects

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Abstract

In this paper, the global exponential convergence of T–S fuzzy complex-valued neural networks with time-varying delays and impulsive effects is discussed. By employing Lyapunov functional method and matrix inequality technique, we analyze a type of activation functions with Lipschitz function, and sufficient conditions in terms of complex-valued linear matrix inequality are obtained to ensure the global exponential convergence. Moreover, the framework of the exponential convergence ball in the state space of the considered neural networks and the exponential convergence rate index are also given out. Here, the existence and uniqueness of the equilibrium points need not be considered and the results improve existing results on the Lyapunov exponential stability as special cases. Finally, one numerical example with simulations is given to illustrate the effectiveness of our theoretical results.

Keywords: Complex-valued neural network; Convergence; T–S fuzzy model; Time-varying delay; Impulsive effect; Complex-valued linear matrix inequality

1. Introduction

In the past decades, stability analysis of various classes of neural network models, such as Hopfield neural network, Cohen–Grossberg neural network, cellular neural network and BAM neural network, has been extensively investigated due to their extensive applications in many fields such as pattern recognition, intelligent robot, predictive estimate, biology, signal and image processing. However, in electronic and biological neural systems, the time delays inevitably exist due to the communication between the neurons. Therefore, it is of significant importance to consider stability analysis of neural networks with time delays. Recently, so much attention have been paid to search sufficient conditions to verify the asymptotical or exponential stability of neural networks with delays [1–6]. Meanwhile, in practical applications, impulsive phenomenon exists universally in various fields such as chemical technology, population dynamics and economics, where the state is changed abruptly at certain moments of time. Examples of impulsive

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phenomena can be also found in automatic control system, artificial intelligence, robotics, etc. Therefore, impulsive neural network model belongs to new category of dynamical systems, which are neither continuous nor discrete ones. It is necessary to consider both impulsive effects and delays in the study of stability of neural networks. Recently, many researchers [7–9] have done extensive works on asymptotic behavior of impulsive neural networks with delays.

However, besides impulse effects and delays, in mathematical modeling of real world problems, we encounter some other inconveniences, for example, the complexity, the approximation and the vagueness. Fuzzy logic systems or neural networks have been proved to be universal approximators, i.e., they can approximate any nonlinear functions. Therefore, fuzzy logic systems and neural networks have been widely adopted for nonlinear systems [10–12]. In recent years, fuzzy logic theory has been efficiently applied to many applications and it is an effective approach to model a complex nonlinear system and deal with its stability. Takagi and Sugeno [13] first introduced fuzzy models and then the T–S fuzzy model is successfully and effectively used in complex nonlinear systems [14]. T–S fuzzy systems are nonlinear systems described by a set of IF-THEN rules. It has been shown that the T–S model method can give an effective way to represent complex nonlinear systems by some simple local linear dynamic systems with their linguistic description. Some nonlinear dynamic systems can be approximated by the overall fuzzy linear T–S models for the purpose of dynamical analysis [15]. In addition, the fuzzy neural networks [16] are also a kind of important neural networks. Recently, the authors [17–21] studied the stability problem of T–S fuzzy neural networks with time-varying delays by using the Lyapunov functional theory and LMI technique. Lagrange exponential stability of T–S fuzzy Cohen–Grossberg neural networks with time-varying delays was also discussed in [22].

On the other hand, as an extension of real-valued neural networks, complex-valued neural networks (CVNNs) with complex-valued state, input, connection weight and activation function become strongly desired because of their practical applications in physical systems dealing with electromagnetic, light, ultrasonic and quantum waves. For example, the XOR problem and the detection of symmetry [23] can not be solved with a single real-valued neuron but be solved by a single complex-valued neuron with the orthogonal decision boundaries, which reveals the potential computational power of complex-valued neurons. However, it was known that the main challenge is the choice of the activation functions in study of properties of CVNNs. As we all know, activation functions can be expressed by separating their real and imaginary parts, some results were given for various CVNNs in this way [24–28]. On the other hand, when the activation functions can not be separated into their real and imaginary parts, some stability criteria of CVNNs were also obtained under Lipschitz continuity condition in the complex domain [29–33].

Because there is no equilibrium point, chaos attractor, periodic state or almost periodic state in the neural networks outside the globally attracting set [34], many scholars have studied globally attractive sets of neural networks [35–40]. However, to the best of our knowledge, there is hardly any paper that considers the global convergence of fuzzy complex-valued neural networks with time-varying delays and impulsive effects. These constitute the motivation for the present research.

Motivated by the above analysis, we will generalize the ordinary T–S fuzzy models to express a class of CVNNs with time-varying delays and impulsive effects. The main purpose of this paper is to study the global exponential convergence for T–S fuzzy CVNNs with time-varying delays and impulsive effects, not discussing the existence and uniqueness of the equilibrium point. The main contributions of this paper are the following aspects: (i) The present paper is one of the first papers that attempts to study the global exponential convergence of T–S fuzzy CVNNs with time-varying delays and impulsive effects. (ii) The activation functions have not be separated into their real and imaginary parts. (iii) The established sufficient conditions are expressed in terms of complex-valued linear matrix inequalities to ensure global exponential convergence of T–S fuzzy CVNNs, which can be checked numerically using the effective Yalmip toolbox in Matlab and the convergence ball domains for the discussed networks are also given. (iv) Compared with the existed results, our results include the results in [33] as special cases and are more general.

The rest of this paper is organized as follows: Section 2 describes some preliminaries including some necessary definitions and lemmas. The main results are obtained in Section 3. In section 4, one numerical example is given to confirm the validity of our results. Finally, concluding remarks are presented in Section 5.

Notation: Throughout this paper, the superscripts Q^{-1}, Q^T and Q^* stand for the inverse, transpose and conjugate transpose of matrix Q, respectively. $\mathbb{C}$ is the set of complex numbers. The notation $X > Y$ means that X and Y are Hermitian matrices and that $X - Y$ is positive definite. $\mathbb{C}^n$ and $\mathbb{C}^{n\times n}$ denote, respectively, the set of all n-dimensional complex-valued vectors and all $n \times n$ complex-valued matrices. i shows the imaginary unit, i.e., $i = \sqrt{-1}$. $|a|$ denotes the module of $a \in \mathbb{C}$, and $\|z\|$ denotes the norm of $z \in \mathbb{C}^n$, i.e., $\|z\| = \sqrt{z^* z}$. $\lambda_{\max}(\lambda_{\min})$ refers to maximal(minimal) eigenvalue. N_+ stands for the set of positive integers. Let $\Gamma = \{1, 2, \dots, n\}$.

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