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Short communication

A short note on fuzzy relational inference systems ☆

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Abstract

This paper is a short note contribution to the topic of fuzzy relational inference systems and the preservation of their desirable properties. It addresses the two main fuzzy relational inferences – compositional rule of inference (CRI) and the Bandler–Kohout subproduct (BK-subproduct) – and their combination with two fundamental fuzzy relational models of fuzzy rule bases, namely, the Mamdani–Assilian and the implicative models.

The goal of this short note article is twofold. Firstly, we show that the robustness related to the combination of BK-subproduct and implicative fuzzy rule base model was not proven correctly in [24]. However, we will show that the result itself is still valid and a valid proof will be provided. Secondly, we shortly discuss the preservation of desirable properties of fuzzy inference systems and conclude that neither the above mentioned robustness nor any other computational advantages should automatically lead to a preference of the combinations of CRI with Mamdani–Assilian models or of the BK-subproduct with the implicative models.

Keywords: Fuzzy relational inference systems; Compositional rule of inference; Bandler–Kohout subproduct; Fuzzy relational equations; Robustness of inference

1. Introduction

This paper is a short note contribution to the theme of fuzzy rule based systems, in particular, to the fuzzy relational inference systems.[1] We assume readers to be familiar with the main notions and we recall only the basic facts in order to introduce the problem and the findings.

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[1] The authors are fully aware of other approaches, mainly, e.g., Similarity Based Reasoning [22]. It is also worth mentioning that under certain conditions, an equivalent fuzzy relation based description of some of these inference systems can be built, see [3], so the outcomes of this article are still also highly related to these other systems.

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Let X, Y be non-empty universes and the sets of all fuzzy sets on $X(Y)$ be denoted by $\mathcal{F}(X)$ ($\mathcal{F}(Y)$). A fuzzy rule base is a set of n fuzzy rules in the following form:

$$\textbf{IF } x \text{ is } A_i \quad \textbf{THEN} \quad y \text{ is } B_i\ , \quad i = 1, \dots, n\ , \tag{1}$$

where the fuzzy sets $A_i \in \mathcal{F}(X)$ and $B_i \in \mathcal{F}(Y)$, represent some properties. It is modeled by a fuzzy relation from $\mathcal{F}(X \times Y)$ and two types of fuzzy relations have become standard, namely the Mamdani–Assilian model [13] $\check{R} \in \mathcal{F}(X \times Y)$ and the implicative model $\hat{R} \in \mathcal{F}(X \times Y)$:

$$\check{R}(x, y) = \bigvee_{i=1}^{n} (A_i(x) * B_i(y))\ , \quad x \in X,\ y \in Y\ , \tag{2}$$

$$\hat{R}(x, y) = \bigwedge_{i=1}^{n} (A_i(x) \to B_i(y))\ , \quad x \in X, y \in Y\ , \tag{3}$$

where $\bigvee, \bigwedge$ represent max and min operations respectively.

The models employ appropriate operations representing a conjunctor $*$ and a fuzzy implication $\to$. It makes sense to consider some restriction to arbitrary yet meaningful and rich classes of operations, e.g., left-continuous t-norms and their residual operations.

Such a restriction leads to a complete residuated lattice $\mathcal{L} = ([0, 1], \wedge, \vee, *, \to, 0, 1)$ as the underlying algebraic structure [4] and consequently many rich and well investigated properties can be employed, see [18] – a few that will be required in the sequel are listed below: For any $a, b, c \in \mathcal{L}$, we have

$$a \to b \geq a\ , \tag{4}$$

$$a \to b \leq a \to c \text{ whenever } b \leq c\ , \tag{5}$$

$$a \to (b \to c) = (a * b) \to c = (b * a) \to c\ , \tag{6}$$

$$\bigvee_{i \in \mathcal{I}} a_i \to b = \bigwedge_{i \in \mathcal{I}} (a_i \to b)\ , \tag{7}$$

$$(a \to b) * c \leq a \to (b * c) \tag{8}$$

where $\bigvee, \bigwedge$ represent sup, inf, respectively, over the complete lattice $\mathcal{L}$.

The difference between both models have been the point of interest of many investigations. We only provide readers with some relevant references [2,7,17].

Given an observation $A' \in \mathcal{F}(X)$, the inferred output $B' \in \mathcal{F}(Y)$ is obtained with the fuzzy *inference mechanism* and the "chosen" fuzzy rule base model R. For fuzzy relational models, the inference mechanisms uses an image of a fuzzy set A' under a fuzzy relation R as follows:

$$B' = A' @ R\ , \tag{9}$$

where the image @ is derived from a particular fuzzy relational composition. For the sake of simplicity, we will not distinguish between images and compositions. The most commonly employed compositions @ are: the sup $-*$ composition (denoted by $\circ$), that determines the *Compositional Rule of Inference* (CRI) [26]; and the inf $- \to$ composition (denoted by $\lhd$) that determines the *Bandler–Kohout subproduct* [1]. The inferred output B' is then determined in one of the following ways:

$$(A' \circ R)(y) = \bigvee_{x \in X} (A'(x) * R(x, y))\ , \quad y \in Y\ , \tag{10}$$

$$(A' \lhd R)(y) = \bigwedge_{x \in X} (A'(x) \to R(x, y))\ , \quad y \in Y\ , \tag{11}$$

where $\bigvee, \bigwedge$ represent sup, inf, respectively, over the non-empty (possibly infinite) domain X.

Note, that whilst the CRI has become the standard since its beginning, the very first use of the Bandler–Kohout subproduct (BK-Subproduct, for short) as an inference mechanism has been in [19] and it has attracted much more attention in the recent years [24,12,14,23,25].