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Fuzzy inference systems preserving Moser–Navara axioms

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Abstract

Satisfiability of three naturally proposed axioms, which are desirable for fuzzy inference system, was discussed by Moser and Navara in [36]. The fuzzy rule base models considered in this study were mainly the Mamdani–Assilian ones and partly also logically motivated implicative fuzzy systems. The investigation leads to the fact that often none of these fuzzy systems do satisfy the given axioms simultaneously. Therefore, Moser and Navara proposed so-called *conditionally firing rules* and proved that the simultaneous satisfaction of the three axioms is feasible under very mild and practically non-restrictive conditions. Note that the inference mechanism, tailored to the fuzzy rules in the investigation, was the compositional rule of inference (abb. CRI) and similarly, the concept of conditionally firing rules were stemming from the CRI and the Mamdani–Assilian model of a fuzzy rule base. For the implicative interpretation of fuzzy rules, the original axioms proposed by Moser and Navara were later on re-defined by Štěpnička and Mandal with the aim to express the meaning of the original axioms initially proposed for the Mamdani–Assilian rules. Furthermore, conditionally firing implicative rules were proposed based on the so-called Bandler–Kohout subproduct (abb. BKS) inference mechanism and a study similar to the one by Moser and Navara was undertaken for them. The study encompasses the satisfaction of the modified axioms for the use of implicative rules and the BKS. Furthermore, some additional results confirming the applicability of the conditionally firing rules satisfying the Moser–Navara axioms are also provided, including the universal approximation ability. However, the solvability criterions for fuzzy relational equations give a strong preference to the usage of the other combinations, namely, to the combination of CRI with the implicative interpretation of fuzzy rules and to the combination of BKS with the Mamdani–Assilian interpretation of fuzzy rules. Therefore, also these combinations of fuzzy inference mechanisms and interpretations of fuzzy rules should be investigated with respect to the satisfaction of Moser–Navara axioms. The main purpose of this article, which extends the short conference contribution [47] by Štěpnička, is to provide such an investigation jointly with the review on conditionally firing rules and supplementary results.

Keywords: Moser–Navara axioms; Fuzzy relational inference systems; Compositional rule of inference; Bandler–Kohout subproduct; Fuzzy relational equations; Correctness of inference; Universal approximation property

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1. Introduction

Fuzzy inference systems are systems for deriving meaningful outputs from imprecise inputs based on some fuzzy rule base. Many kinds of fuzzy inference systems have been studied, so we will only refer interested readers to the relevant literature [3,14,31,54]. This work stems from probably the most fundamental fuzzy inference systems that were already proposed by Zadeh – from the so-called fuzzy relational inference (FRI) systems. In such systems, a fuzzy rule base is modeled/interpreted by a single fuzzy relation and the fuzzy inference mechanism is modeled by an image of a fuzzy set under the fuzzy relation that interprets the fuzzy rule base.

Let the restriction to this type of system be not viewed as a lack of respect to researchers dealing with many other fuzzy inference systems and the but rather as a reasonable restriction that is usually necessary in order to obtain sufficiently significant results for a narrowed class of systems. Indeed, for example Similarity-based reasoning [15,19] and related approaches [46] unquestionably proved their practical usefulness. On the other hand, they are not unrelated to the fuzzy relational inference systems [7] and thus, under some specific settings of particular components of such systems. And the same may be stated also about seemingly not so related fuzzy inference systems such as, e.g., the Perception-based Logical Deduction [40,18] that may also under some assumptions be studied as a specific fuzzy relational inference system [17]. Therefore, the conclusions provided in this article hold also for many other yet related systems.

1.1. Fuzzy rules in fuzzy relational inference systems

Let us consider a fuzzy rule base describing a dependency of elements of a non-empty universe Y on the elements of a non-empty universe X. The information of the fuzzy rule base is hidden in pairs of antecedent and consequent fuzzy sets $(A_1, B_1), \dots, (A_n, B_n)$, expressing that fuzzy set $B_i \in \mathcal{F}(Y)$ is assigned to fuzzy set $A_i \in \mathcal{F}(X)$ and where, as usually, $\mathcal{F}(X)$ and $\mathcal{F}(Y)$ denote the sets of fuzzy sets on X and Y, respectively.

The first of two standard approaches to modeling a given fuzzy rule base by a fuzzy relation on $X \times Y$ consists in constructing the following fuzzy relation

$$\hat{R}(x, y) = \bigwedge_{i \in N_n} (A_i(x) \to B_i(y)) \tag{1}$$

where $\to$ is a (preferably residual) fuzzy implication [2] and $N_n = \{1, \dots, n\}$. Let us recall the description of the meaning of such a model of fuzzy rules, provided by Dubois et al. in [16]. The authors claim that each fuzzy rule is viewed as a constraint, which naturally leads to a conjunctive way of merging the individual constraints since the more constraints and the less possible values to satisfy them. The used fuzzy implication naturally represents the logical operation IF–THEN and the minimum naturally represents the logical conjunction and thus we argue that the fuzzy relation $\hat{R}$ is a proper model of conjunctively merged IF–THEN fuzzy rules [6].

The other standard approach, and to be frank much more often used, follows the seminal work of Mamdani and Assilian [32] and models the fuzzy rule base with the following fuzzy relation

$$\check{R}(x, y) = \bigvee_{i \in N_n} (A_i(x) \otimes B_i(y)) \tag{2}$$

where $\otimes$ denotes a (preferably left-continuous) t-norm.

The fuzzy relation $\check{R}$ is not primarily built as a mathematical model of conditional IF–THEN sentences encoded in the fuzzy rules but it rather employs the Cartesian product of antecedent and consequent fuzzy sets, which is also explicitly stated in the original work [32]. We again recall the work of Dubois et al. [16] where the authors claim that fuzzy rules modeled by $\check{R}$ are, unlike in the above constrained view, considered as pieces of data that are naturally accumulated by the maximum in (2) for the aggregation of rules.

Although the above discussed approaches to modeling fuzzy rule bases belong to crucial topics, it is not the main goal of this article to discuss the advantages and differences between them. Therefore, we only provide readers with references to the relevant literature [9,10,16,20,25,39,41,48].

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