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A comprehensive study of fuzzy covering-based rough set models: Definitions, properties and interrelationships

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Abstract

Fuzzy covering-based rough set models are hybrid models using both rough set and fuzzy set theory. The former is often used to deal with uncertain and incomplete information, while the latter is used to describe vague concepts. The study of fuzzy rough set models has provided very good tools for machine learning algorithms such as feature and instance selection. In this article, we discuss different types of dual fuzzy rough set models which all consider fuzzy coverings. In particular, we study two models using non-nested level-based representation of fuzziness. In addition to the study of the theoretical properties for each model, interrelationships between the different models are discussed, resulting in a Hasse diagram of fuzzy covering-based rough set models for a finite fuzzy covering, an IMTL-t-norm and its residual implicator.

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1. Introduction

Rough set theory was introduced by Pawlak in 1982, as a tool to deal with uncertainty caused by indiscernibility and incompleteness in information systems [20]. To discern the elements of a universe U , an equivalence relation on U is considered. Pawlak's definition appeared to have many equivalent formulations which are mutually interpretable [34]. Hence, it is possible to consider the element-based definition using the equivalence relation, the granule-based definition using the partition U/E or the subsystem-based definition using the σ -algebra over U/E .

All equivalent formulations can be generalized. A first generalization of rough sets is obtained by replacing the equivalence relation by a general binary relation or by a neighborhood operator [26,27,33,40]. In this case, the binary relation or the neighborhood operator determines collections of sets which no longer form a partition of U . A second generalization is derived when we substitute the partition obtained by the equivalence relation with a covering,

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i.e., a collection of non-empty sets such that its union is equal to U . Żakowski proposed the first notion of covering-based rough set approximation operators in 1983 [36]. However, his approximation operators are no longer dual as in Pawlak's case. For this reason, Pomykała [21] studied the operators of Żakowski and their dual operators. The pair consisting of the lower approximation operator of Żakowski and its dual upper approximation operator is called the tight pair, while the pair consisting of the upper approximation operator of Żakowski and its dual lower approximation operator is called the loose pair [4]. Finally, to generalize the subsystem-based definition, a closure system over U , i.e., a family of subsets of U that contains U and is closed under set intersection, can be considered [34].

Already in 1965 fuzzy set theory was introduced by Zadeh [35] to describe vague concepts and from early on, it has been clear that rough set theory is complementary rather than competitive with it. The vestiges of fuzzy rough set theory date back to the late 1980s, and originate from work by Fariñas del Cerro and Prade [12], Dubois and Prade [11], Nakamura [19] and Wygalak [30]. From 1990 onwards, research on the hybridization between rough sets and fuzzy sets has flourished. An extensive overview of fuzzy relation-based rough set models can be found in [5].

Analogously as in the crisp case, fuzzy binary relations are closely related with fuzzy neighborhood operators. Therefore, in this paper we will work with fuzzy neighborhood-based rough set models instead of fuzzy relation-based rough sets. In particular, we will focus on models using fuzzy neighborhood operators which are constructed using a fuzzy covering [8,18].

Besides fuzzy extensions of the element-based rough set models, we will discuss fuzzy extensions of the granule-based rough set models. Fuzzy extensions of the tight covering-based approximation operators have been studied by Li et al. [17], Inuiguchi et al. [15,16] and Wu et al. [29] which we resume here. Moreover, in [6], two tight fuzzy covering-based rough set models were presented: one using non-nested representation by levels introduced by Sánchez et al. [25] and one constructed from an intuitive point of view. In addition, we recall the loose fuzzy covering-based rough set model of Li et al. [17] and introduce a new loose fuzzy covering-based rough set model using representation by levels. To our knowledge, the tight and loose fuzzy covering-based rough set models discussed in this article are all the fuzzy granule-based models currently available.

The goal of this article is to provide an overview of the research on fuzzy covering-based rough set models, for which we keep applications in mind. For every model, we study different theoretical properties which are meaningful for machine learning applications. Moreover, we compare different models with respect to each other by discussing the accuracy of approximation operators. For every application, there is a necessary trade-off between the different aspects of a fuzzy rough set model, such as the satisfied properties and the accuracy of the approximation operators. To this aim, the article provides a theoretical background for researchers to use as a starting point in finding a suitable model for their particular application.

The outline of the article is as follows: in Section 2, we discuss some preliminary results. First, concepts of covering-based rough set theory are discussed. Next, we discuss different fuzzy neighborhood operators based on a fuzzy covering. Furthermore, the technique of representation by levels is discussed. To end the preliminaries, we discuss different properties of fuzzy covering-based rough sets. In Section 3, an overview of fuzzy covering-based rough set models is provided. Besides fuzzy neighborhood-based rough set models, we discuss fuzzy extensions of the tight and loose covering-based approximation operators. In Section 4, we study the interrelationships between the different models. We construct a Hasse diagram for a finite fuzzy covering, an IMTL-t-norm and its residual implicator. Conclusions and future work are stated in Section 5.

Finally, note that this paper extends the conference paper [6], where a limited part of the results we obtain was presented.

2. Preliminaries

Throughout this paper we assume that the universe of discourse U is a non-empty, possibly infinite set of objects. We first recall some notions on crisp covering-based rough sets. Furthermore, we discuss fuzzy neighborhood operators based on a fuzzy covering. In addition, we study the technique of representation by levels, a technique constructed to describe fuzzy concepts with non-nested crisp representatives. To end this preliminary section, we discuss some properties a fuzzy covering-based rough set model can satisfy.

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