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On the construction of radially symmetric copulas in higher dimensions

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Received 18 October 2016; received in revised form 8 August 2017; accepted 2 November 2017

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Abstract

We prove a representation theorem for copulas that are (simultaneously) symmetric and radially symmetric. We use this representation theorem to propose a method to construct an n-ary symmetric function that is radially symmetric, starting from an $(n-1)$-dimensional copula and an n-ary auxiliary function. We study the necessary and sufficient conditions on this auxiliary function that guarantee our construction method to result in a symmetric and radially symmetric n-dimensional copula. We examine several options for defining the auxiliary function in the trivariate case, inspired by the nesting of copulas, the lifting of copulas and product-like extensions. For each choice of auxiliary function, we provide several examples for different families of copulas.
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Keywords: Copula; Radial symmetry; Symmetry; Aggregation function

1. Introduction

The concept of symmetry of a random variable is uniquely defined. A random variable X is said to be symmetric about a if $X-a$ has the same distribution as $a-X$. In the multivariate case, the situation is more complicated, as there are several ways to generalize the notion of univariate symmetry (see, for example, [44]). One such possible generalization is the concept of radial symmetry. An n-dimensional random vector $(X_1,\ldots,X_n)$ is said to be radially symmetric about $(a_1,\ldots,a_n)$ if the random vector $(X_1-a_1,\ldots,X_n-a_n)$ has the same distribution as the random vector $(a_1-X_1,\ldots,a_n-X_n)$.

One of the advantages of radial symmetry is that it is a non-parametric property [36] and, as a consequence, it can be studied on the basis of the associated n-copula of the random vector. Recall that an n-dimensional copula (or, for short, n-copula) is a multivariate distribution function with the property that all of its n univariate marginals are uniform distributions on $[0,1]$, i.e., an n-copula is a $[0,1]^n \to [0,1]$ function that satisfies the following conditions:

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https://doi.org/10.1016/j.fss.2017.11.004
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(1) $C_n(\mathbf{x}) = 0$ if $\mathbf{x}$ is such that $x_i = 0$ for some $i \in \{1, 2, ..., n\}$.
(2) $C_n(\mathbf{x}) = x_i$ if $\mathbf{x}$ is such that $x_j = 1$ for all $i \neq j$.
(3) C_n is n-increasing, i.e., for any n-box $\mathbf{P} = \times_{i=1}^{n}[a_i, b_i] \subseteq [0, 1]^n$ it holds that

$$V_{C_n}(\mathbf{P}) = \sum_{\mathbf{x} \in \text{vertices}(\mathbf{P})} (-1)^{S(\mathbf{x})} C_n(\mathbf{x}) \geq 0 ,$$

where $S(\mathbf{x}) = \#\{i \in \{1, 2, ..., n\} \mid x_i = a_i\}$.

$V_{C_n}(P)$ is called the C_n-volume of P.

Remark 1. When $n - k$ of the arguments of an n-copula are set equal to 1, we obtain a k-dimensional marginal of the n-copula, which itself is a k-copula.

For a given n-copula C_n, $\hat{C}_n$ denotes the associated survival n-copula, and is given by

$$\begin{aligned}\hat{C}_n(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i - (n-1) + \sum_{i<j}^{n} C_n(1, \ldots, 1 - x_i, \ldots, 1 - x_j, \ldots, 1) \\ - \sum_{i<j<k}^{n} C_n(1, \ldots, 1 - x_i, \ldots, 1 - x_j, \ldots 1 - x_k, \ldots, 1) + \ldots \\ + (-1)^n C_n(1 - x_1, 1 - x_2, \ldots, 1 - x_n) .\end{aligned} \quad (1)$$

Survival copulas have a clear probabilistic interpretation: if the random vector $(U_1, \ldots U_n)$ has the copula C_n as its joint distribution function, then $\hat{C}_n$ is the joint distribution function of the random vector $(1 - U_1, \ldots, 1 - U_n)$. This probabilistic interpretation has led to several studies of the transformations of copulas that are induced by certain types of transformations on random variables [16,17,26]. These transformations have been generalized and studied in the framework of aggregation functions. For example, such transformations were studied in [7,9] for bivariate aggregation functions, while a specific transformation was studied in [11] for higher dimensions.

Due to Sklar's theorem, which states that any continuous multivariate distribution function can be written in terms of its n univariate marginals by means of a unique n-copula, n-copulas have become one of the most important tools for the study of certain types of properties of random vectors, such as stochastic dependence (see [14,35] for more details on n-copulas).

As mentioned before, an example of a property that can be directly studied using copulas is the property of radial symmetry. It can be easily shown that a continuous random vector $(X_1, \ldots, X_n)$ is radially symmetric about $(a_1, \ldots, a_n)$ if and only if, for any $j \in \{1, \ldots, n\}$, $X_j - a_j$ has the same distribution as $a_j - X_j$, and $C_n = \hat{C}_n$, where C_n is the copula associated to the random vector $(X_1, \ldots, X_n)$. Due to this characterization, we say that an n-copula C_n is radially symmetric if it satisfies the identity $C_n = \hat{C}_n$. Clearly, radial symmetry of an n-copula implies the radial symmetry of its lower dimensional marginals, however, the converse statement is not true.

It is important to remark that the concept of radial symmetry of an n-copula is different from the concept of symmetry. Symmetric copulas are the copulas associated to exchangeable random variables. Formally, we say that an n-copula C_n is symmetric if for any permutation σ of $\{1, 2, \ldots, n\}$ and for any $\mathbf{x} \in [0, 1]^n$ it holds that

$$C_n(x_1, x_2, \ldots, x_n) = C_n(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)}) .$$

Note that if an n-copula is symmetric, all of its k-dimensional marginals coincide for any $k \in \{2, \ldots n - 1\}$.

Radially symmetric copulas have a particular importance in stochastic simulation and statistics, as they can be used, in certain situations, in the multivariate version of the antithetic variates method, which is a variance reduction technique used in Monte Carlo methods [29]. Additionally, there has also been a growing interest in developing statistical tests for testing the presence of radial symmetry [1,3,10,18,22,38,41].

In the bivariate case, well-known examples of families of copulas that are radially symmetric are the Frank family and the Farlie–Gumbel–Morgenstern (FGM) family [35]. However, there are only a few families of n-copulas that are radially symmetric for $n \geq 3$, elliptical copulas being the best known [29].

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Please cite this article in press as: J.J. Arias-García et al., On the construction of radially symmetric copulas in higher dimensions, Fuzzy Sets Syst. (2017), https://doi.org/10.1016/j.fss.2017.11.004

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