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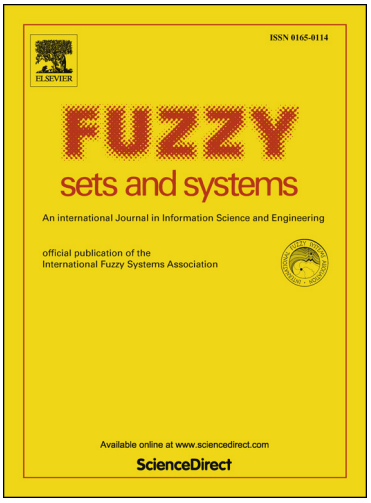

Convolution lattices

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Convolution lattices

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Abstract

We propose two convolution operations on the set of functions between two bounded lattices and investigate the algebraic structure they constitute, in particular the lattice laws they satisfy. Each of these laws requires the restriction to a specific subset of functions, such as normal, idempotent or convex functions. Combining all individual results, we identify the maximal subsets of functions resulting in a bounded lattice, and show this result to be equivalent to the distributivity of the lattice acting as domain of the functions. Furthermore, these lattices turn out to be distributive as well. Additionally, we show that for the larger subset of idempotent functions, although not satisfying the absorption laws, the convolution operations satisfy the Birkhoff equation.

Keywords: Algebra, Convolution operations, Lattice

1. Introduction

The mathematical operation of convolution and related operations play a pivotal role in science, engineering and mathematics [1, 2]. In the standard setting, convolution takes two real functions as input and outputs a third real function that represents the integral of the pointwise multiplication of the two functions as a function of the amount that one of the original functions is translated. More formally, given two real functions f and g, their convolution is the function $f * g$ defined by

$$(f * g)(t) = \int f(\lambda)g(t-\lambda)\mathrm{d}\lambda\,.$$

The convolution operation has applications in probability and statistics, differential equations, signal processing, natural language processing, image pro-

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