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Algorithmic and logical characterizations of bisimulations for non-deterministic fuzzy transition systems

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Abstract

Bisimulation is a well-known behavioral equivalence for discrete event systems and has been developed in fuzzy systems quickly. In this paper, we adopt an approach of the relational lifting that is one of the most used methods in the field of bisimulation research, to define it for a non-deterministic fuzzy transition system. An $\mathcal{O}(|S|^4|{\rightarrow}|^2)$ algorithm is given for testing bisimulation where $|S|$ is the number of states and $|{\rightarrow}|$ the number of transitions in the underlying transition systems. Two different modal logics are presented. One is two-valued and indicates whether a state satisfies a formula, which is an extension of Hennessy–Milner logic. The other is real-valued and shows to what extent a state satisfies a formula. They both characterize bisimilarity soundly and completely. Interestingly, the second characterization holds under a class of fuzzy logics. In addition, this real-valued logic allows us to conveniently define a logical metric that captures the similarity between states or systems. That is, the smaller distance, the more states alike. Although the work is inspired by the corresponding work in probabilistic systems, it is obviously different. In particular, the real-valued logic in this paper remains unexplored in fuzzy systems, even in probabilistic systems.

Keywords: Bisimulation; Fuzzy transition system; Modal logic; Logical characterization; Logical metric

1. Introduction

Bisimulations are established forms of behavioral equivalences for discrete event systems like process algebras, Petri nets or automata models and have been widely used in many areas of computer science. They are helpful to model-checking by reducing the number of states of systems.

Bisimulations have attracted much attention from researchers who work in the field of fuzzy systems and have been developed quickly [3,5–7,9,15,21,31,33]. The bisimulation of Cao et al. [5] is defined on an equivalence relation. This paper generalizes it to a general case. In this setting, we require that if (s, t) be a pair of states in a simulation relation

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R, i.e., sRt, then t can mimic all the stepwise behaviors of s with respect to R. Thus, if s can perform an action and evolve into a distribution μ, then t can perform the same action to another distribution ν such that ν can somehow simulate the behavior of μ according to R. To formalize the mimicking of μ by ν, we have to lift R to the relation $R^\dagger$ between distributions and require $\mu R^\dagger \nu$. We say that R is a bisimulation provided that R^{-1}, the inverse of R, is also a simulation.

A good scientific concept is often elegant, even seen from many different perspectives. Bisimulation is one of such concepts in classical and probabilistic systems, as it can be characterized in a great many ways such as fixed point theory, modal logics, game theory, coalgebras etc. We believe that (fuzzy) bisimulation is also one of such concepts in fuzzy systems. For example, Cao et al. [5] used the fixed point to characterize bisimulations, while Wu and Chen [32] characterized them by using coalgebras. We will provide in this paper two characterizations, from the perspectives of decision algorithms and modal logics.

The algorithm tests whether two states are bisimilar. It is realized through deciding whether they are in the greatest bisimulation (bisimilarity) that can be approached by a family inductively defined relations (see Definition 4.1). This algorithm is inspired by Baier in [1] but obviously different. Baier decided whether two (probability) distributions are related by some lifted relation in terms of the maximum flow algorithm, while we adopts a simple but subtle algorithm to do this for two (possibility) distributions (see Algorithm 1). The time complexity of the algorithm determining bisimulation is $\mathcal{O}(|S|^4|{\rightarrow}|^2)$ where $|S|$ is the number of states and $|{\rightarrow}|$ the number of transitions in the underlying transition systems.

Because of connections between modal logics and bisimulations, whenever a new bisimulation is proposed, the quest starts for the associated logic, such that two states or systems are bisimilar iff they satisfy the same modal logical formulae. Along this line, a great amount of work has appeared that characterizes various kinds of classical (or probabilistic) bisimulations by appropriate logics, e.g. [8,10,13,14,17,18,22,27,29,34]. Although, bisimulations have been investigated extensively in fuzzy systems, there is little work about the connections between (fuzzy) bisimulations and modal logics. Fan in [15] characterized (fuzzy) bisimulations for fuzzy Kripke structures in terms of Gödel modal logic; Wu and Deng in [31] characterized bisimilarity for (deterministic) fuzzy transition systems by using a fuzzy style Hennessy–Milner logic.

Another work of this paper is to characterize bisimilarity for (nondeterministic) fuzzy transition systems. We first give a Hennessy–Milner style modal logic. This logic is two-sorted and has state formulae and distribution formulae, which is different from the logic in [31] only including state formulae. It is two-valued in the sense whether a state satisfies a formula or not. We also provide a real-valued modal logic that shows to what extent a state satisfies a formula. To the best of our knowledge, this real-valued logic remains unexplored in the literature. Both these two logics characterize bisimilarity soundly and completely. Interestingly, the second characterization holds under a class of fuzzy logics. It should also be pointed out that two-valued logical characterization is motivated by the corresponding work in probabilistic systems. However, the proof in fuzzy case is more difficult than that in probabilistic case, see Theorem 5.2 and Remark 5.4 for details. Finally, with the help of the real-valued logic, we define a logical metric that is a more robust way of formalizing similarity between fuzzy systems than bisimulations. The smaller the logical distance, the more states behave similarly. In particular, the logical distance between two states is 0 iff they are exactly bisimilar.

The rest of this paper is structured as follows. We briefly review some basic concepts used in this paper in Section 2. Section 3 introduces the notions of lifting operation and bisimulation. Some properties about them are discussed. Moreover, an algorithm is given for determining whether two distributions are related by some lifted relation. In Section 4, we present an algorithm for testing bisimulation. In the subsequent section, we provide a two-valued and a real-valued logics, respectively. They both characterize bisimilarity soundly and completely. In Section 6, we define a logical metric to measure the similarity between states or systems. Finally, this paper is concluded in Section 7 with some future work.

2. Preliminaries

In this section, we briefly recall some notions used in this paper.

The notions about fuzzy set are mainly borrowed from [5]. Let S be a set and μ a *fuzzy set* in S. The *support* of μ is the set $\mathrm{supp}(\mu) = \{s \in S \mid \mu(s) > 0\}$. We denote by $\mathcal{F}(S)$ the set of all fuzzy sets in S. Whenever $\mathrm{supp}(\mu)$ is a finite set, say $\{s_1, s_2, \cdots, s_n\}$, then a fuzzy set μ can be written in Zadeh's notation as follows:

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