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Deviation-based aggregation functions

Marián Decký [a], Radko Mesiar [a,b], Andrea Stupňanová [a,*]

[a] *Slovak University of Technology in Bratislava, Faculty of Civil Engineering, Radlinského 11, 810 05 Bratislava, Slovak Republic*
[b] *Institute for Research and Applications of Fuzzy Modeling, University of Ostrava, CE IT4Innovations, 30. dubna 22, 701 03 Ostrava, Czech Republic*

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Abstract

Based on the original idea of Daróczy, deviation functions expressing the deviation of two real values x and y are generalized into (basic) moderate deviation functions. Subsequently, a new type of deviation-based aggregation function is introduced, studied and exemplified. Our approach generalizes, among others, several types of mixture operators, and can be seen as an alternative to penalty-based constructions of aggregation functions.

Keywords: Aggregation function; Deviation function; Mean; Moderate deviation function; Penalty function

1. Introduction

Constructions of aggregation functions related to some well-defined compensation of replacing the n-tuple of aggregated input values by a single output known from the literature are mostly based on minimization of some penalty function [15,7,3,4]. On the other hand, in the framework of functional equations, deviation functions [8,9] were considered to define and study some particular means (not necessarily monotone). Observe that though in some cases both approaches yield the same mean, in general they differ. The aim of this contribution is to introduce deviation-based aggregation functions, i.e., to deal with monotone solutions of deviation-based functions. This alternative approach can be seen as an efficient modification of penalty-based approaches, and it also allows to introduce weights even for non-symmetric aggregation functions, similarly as this was stressed in the case of penalty-based case related to dissimilarity functions of Calvo et al. [7].

The paper is organized as follows. In the next section we recall penalty-based constructions. Then, in Section 3, deviation-based construction for symmetric aggregation functions is introduced and some basic examples are added. A deeper discussion of general deviation-based aggregation functions is given in Section 4, including the introduction of weights. Finally, some concluding remarks are added.

* Corresponding author.
E-mail addresses: marian.decky@stuba.sk (M. Decký), radko.mesiar@stuba.sk (R. Mesiar), andrea.stupnanova@stuba.sk (A. Stupňanová).

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2. Penalty-based aggregation functions

The idea of introducing aggregation functions by means of penalty functions is due to Yager [15]. In this section, $I \subset \mathbb{R}$ is a real interval.

Definition 2.1. Local penalty is a function $LP : I^2 \to \mathbb{R}^+$ satisfying for any $u, v, y \in I$ the following requirements:

(i) $LP(u, y) = 0$ if and only if $u = y$;
(ii) $LP(u, y) \geq LP(v, y)$, if $|u - y| \geq |v - y|$.

Then a penalty function $P : I^{n+1} \to \mathbb{R}^+$ is defined, for any $\mathbf{x} \in I^n$ and $y \in I$ as:

$$P(\mathbf{x}, y) = \sum_{i=1}^{n} LP(x_i, y), \tag{1}$$

where $y \in I$ is called the fused value of $\mathbf{x} \in I^n$.

The function P measures the total penalty incurred, whenever y is the fused value of the input vector $\mathbf{x}$. The best (optimum) fused value of the elements in $\mathbf{x}$ is the value y^* that minimizes the penalty function P. Observe that the best fused value y^* need not exist, in general, or there can be several best fused values. Then, if they form an interval, its mid-point is considered. Thus, the best fused value of $\mathbf{x}$, namely, $f(x_1, \ldots, x_n)$ is obtained as the value y^* such that

$$P(\mathbf{x}, y^*) = \min_y P(\mathbf{x}, y).$$

The function f is called by a penalty-based function.

By providing different forms for the penalty functions Yager and Rybalov [16] presented different forms of fusion methods, e.g. for penalty function $P(\mathbf{x}, y) = \sum_i |x_i - y|$ we get median type aggregation.

When assuming that each of the observations has an associated weight function (e.g., the weight can be a measure of the importance or reliability of a certain sensor), the weighted penalty function is obtained as follows:

Definition 2.2. Let $LP : I^2 \to \mathbb{R}^+$ be defined as in previous definition. The weighted penalty function $wP : I^{n+1} \to \mathbb{R}^+$ is defined, for any $\mathbf{x} \in I^n$ and $y \in I$ as:

$$wP(\mathbf{x}, y) = \sum_{i=1}^{n} w_i LP(x_i, y),$$

where $w_i \geq 0$ is the weight associated to the observation x_i, with $i = 1, \ldots, n$.

Also in the later, more general penalty-based approaches, the aggregation of inputs $x_1, \ldots, x_n$ was just the minimizer (or the middle point of minimizers set). Yager's approach was first generalized by Calvo et al. [7], introducing the notion of a dissimilarity function.

Definition 2.3. Let $K : \mathbb{R} \to \mathbb{R}^+$ be a convex function such that $K(x) = 0$ if and only if $x = 0$, and $s : I \to \mathbb{R}$ be a continuous strictly monotone function. Then dissimilarity L is given in the form

$$L(x, y) = K(s(x) - s(y)),$$

and penalty function $P : I^{n+1} \to \mathbb{R}^+$ is defined, for any $\mathbf{x} \in I^n$ and $y \in I$ as:

$$P(\mathbf{x}, y) = \sum_{i=1}^{n} L(x_i, y).$$

A function $f_P : \bigcup_{n \in \mathbb{N}} I^n \to I$, defined for all $\mathbf{x} \in I^n$ and $n \in \mathbb{N}$ by