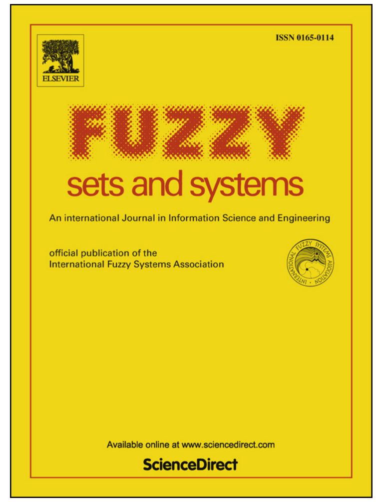
Accepted Manuscript



On general conditions for nestedness of the solution set of fuzzy-interval linear systems

Hana Veiseh, Taher Lotfi, Tofigh Allahviranloo

PII: S0165-0114(17)30279-8
DOI: http://dx.doi.org/10.1016/j.fss.2017.07.004
Reference: FSS 7261

To appear in: *Fuzzy Sets and Systems*

Received date: 19 April 2016
Revised date: 4 May 2017
Accepted date: 1 July 2017

On general conditions for nestedness of the solution set of fuzzy-interval linear systems *

Hana Veiseh [a †] Taher Lotfi [a ‡] Tofigh Allahviranloo [a §]

[a] Department of Applied Mathematics, Hamedan Branch, Islamic Azad University, Hamedan, Iran.

Abstract

This study addresses the problem of providing general conditions for nestedness of the solution set of fuzzy-interval linear systems of equalities and inequalities, relying on the fact that they are a direct extension of the interval linear systems. Some illustrative examples demonstrate an applicable viewpoint of the results, which connects our obtained results with the optimization theory.
keywords: Fuzzy-interval system, Nestedness property, Equality system, Inequality system, Optimization theory.

1 Introduction

There is a wealth of research written on interval or fuzzy linear systems [1–34]. The first article about interval linear systems was written by Oettli and Prager in 1964 [27], later in 1981 Gerlach followed their work by an analogous idea for interval systems of linear inequalities [11]. On the other hand, fuzzy researchers at least by 1980 started working on fuzzy linear systems as a seperate study [2, 5, 7, 10, 13, 15, 17, 21]. Although several methods have been proposed to solve fuzzy linear systems of equations, to the best of our knowledge, there are few known results about fuzzy linear systems of inequalities.

A fuzzy linear system that is not carefully analyzed can lead to a solution that is not a fuzzy-interval. To see this problem, consider, for example, the fuzzy linear system presented by Allahviranloo and Ghanbari in 2011 [2]. They showed that the resultant solution in [10] is not a fuzzy solution. This is because of the two problems with the solutions coming out of the way current fuzzy linear systems are solved. The first problem is that resultant intervals associated with α-levels are not always nested [17]. The second is that when one extends (fuzzy) interval spaces into a space which has inverses, the α-levels coming from solutions may not be proper intervals and it comes down to finding an inconsistent result [17]. An improper interval is one whose left endpoint is larger than the right endpoint, which may occur when fuzzy-intervals are embedded into an algebraic system with inverses (additive and multiplicative) [17]. It is clear that the space of intervals by itself possesses neither additive nor multiplicative inverses and was extended to obtain an algebraic structure with additive/multiplicative inverses, which was known in the interval literature at least by 1980 [14]. Of course, these two problems may occur in both systems of fuzzy equalities and inequalities.

Recently Lodwick and Dubois in [17] studied the properties such as nestedness of the solution set of a system of fuzzy-interval linear equations, and presented a unified approach to solve them. In other words, they developed among other things what they call the epistemic solution of a fuzzy linear system. Following their study, we give some general conditions not only for fuzzy-interval linear equalities but also for fuzzy-interval linear inequalities. Furthermore, we just verify the nestedness of solutions of fuzzy-interval systems. It is worth mentioning, that studying of the nestedness of the solution set of fuzzy-interval linear systems of inequalities has not been considered in the literature yet and it can be considered as the main contribution of this study.

Lack of nestedness property may be caused to many anomalous or inconsistent results in various fuzzy-interval linear system studies [17]. This is another motivation for studying this problem.

A linear system is called solvable if it has a solution, and feasible if it has a nonnegative solution. So feasibility means nonnegative solvability. To characterize solvability and feasibility of a system of equality or

*This research was supported by Islamic Azad University, Hamedan Branch
†hana.veiseh@iauh.ac.ir, veisehana@yahoo.com
‡Corresponding author: lotfi@iauh.ac.ir, lotfitaher@yahoo.com
§tofigh@allahviranloo.com