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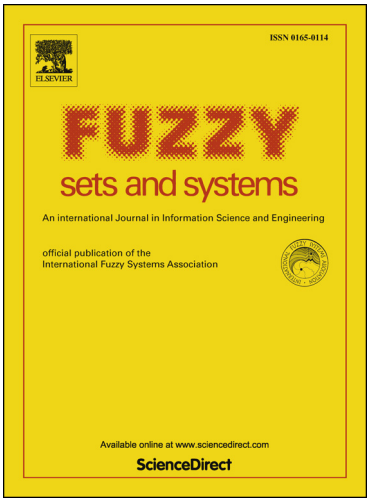

Fuzzy topological spaces with conical neighborhood systems

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Fuzzy topological spaces with conical neighborhood systems[☆]

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Abstract

For a commutative, integral, and meet continuous quantale $\mathcal{Q}$, a CNS space is defined to be a $\mathcal{Q}$-topological space such that the $\mathcal{Q}$-neighborhood system of each point is a conical $\mathcal{Q}$-filter. These spaces reduce to fuzzy T-locality spaces in the case that the quantale is the unit interval endowed with a left continuous t-norm T; and reduce to probabilistic $\mathcal{Q}$-topological spaces in the case that $\mathcal{Q}$ is an MV-algebra. For a continuous quantale $\mathcal{Q}$, a necessary and sufficient condition is obtained for the category of CNS spaces to be simultaneously reflective and coreflective in the category of stratified $\mathcal{Q}$-topological spaces. This result provides an interesting example of the interaction between properties of the quantale $\mathcal{Q}$ and that of $\mathcal{Q}$-topological spaces.

Keywords: Fuzzy topology, Category theory, Quantale, Continuous quantale, $\mathcal{Q}$-order, Prefilter, $\mathcal{Q}$-filter, Conical $\mathcal{Q}$-filter, $\mathcal{Q}$-topological space, CNS space

1. Introduction

Let $\mathcal{Q} = (Q, \&)$ be a commutative and integral quantale. By a $\mathcal{Q}$-topology (or a fuzzy topology) τ on a set X we mean a subset $\tau \subseteq Q^X$ that contains all constant maps from X to Q and is closed with respect to finite meets and arbitrary joins. Let x be a point in a $\mathcal{Q}$-topological space X. A neighborhood of x is a fuzzy set $\lambda \in Q^X$ such that $\lambda^\circ(x) = 1$, where λ° is the interior of λ. The neighborhoods of x form a prefilter $\mathcal{N}_x$ on X (a filter in the lattice $(Q^X, \leq)$), called the neighborhood prefilter of x. Meanwhile, the $\mathcal{Q}$-neighborhood system of x is defined to be the $\mathcal{Q}$-filter

$$\mathfrak{N}_x \colon Q^X \longrightarrow Q,$$

where $\mathfrak{N}_x(\lambda) = \lambda^\circ(x)$. It is clear that

$$\mathcal{N}_x = \{\lambda \in Q^X \mid \mathfrak{N}_x(\lambda) = 1\}$$

and that a $\mathcal{Q}$-topological space X is determined by its $\mathcal{Q}$-neighborhood systems $\{\mathfrak{N}_x\}_{x\in X}$; but, X may not be determined by the neighborhood prefilters $\{\mathcal{N}_x\}_{x\in X}$.

It is natural to ask what kind of $\mathcal{Q}$-topological spaces are determined by the neighborhood prefilters $\{\mathcal{N}_x\}_{x\in X}$? In the case that the underlying lattice of $\mathcal{Q}$ is meet continuous, the class of $\mathcal{Q}$-topological spaces with conical $\mathcal{Q}$-neighborhood systems (CNS spaces for short) provides an answer to this question. Explicitly, a CNS space is a $\mathcal{Q}$-topological space X such that the $\mathcal{Q}$-neighborhood system $\mathfrak{N}_x$ of each point in X is a conical $\mathcal{Q}$-filter in the sense that

$$\mathfrak{N}_x(\lambda) = \bigvee_{\nu\in\mathcal{N}_x} \mathrm{sub}_X(\nu, \lambda)$$

for all $\lambda \in Q^X$, where sub_X is the fuzzy inclusion order on Q^X.

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