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Robust fuzzy structural safety assessment using mathematical programming approach

Di Wu *, Wei Gao, Chen Wang, Sawekchai Tangaramvong, Francis Tin-Loi

School of Civil and Environmental Engineering, The University of New South Wales, Sydney, NSW 2052, Australia

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Abstract

This paper presents a robust safety assessment for engineering structures involving fuzzy uncertainties. Uncertain applied loads and yielding capacities of structural elements are modelled as fuzzy variables with associated membership functions representing possibility distributions. A new computation-orientated methodology, namely the α-level collapse assessment (α-level CA) approach, is developed to provide structural safety profile by constructing membership function of the structural collapse load limit accommodating fuzzy uncertainties. The proposed method firstly utilizes the α-level strategy to transform the fuzzy limit analysis into a series of interval limit analyses. By implementing the concept of robust and optimistic optimizations, a mathematical programming (MP) scheme is proposed to explicitly capture the upper and lower bounds of the collapse load limit at each α-sublevel. Subsequently, the membership function of the collapse load limit is established by using the upper and lower bounds obtained from the series of α-sublevel calculations. The proposed α-level mathematical programming scheme preserves the quality of sharpness of the bounds of collapse load limit at each α-sublevel, which consequently provides a rigorous evaluation on the fuzzy profile of the safety of engineering structures against structural collapse. Numbers of numerical examples, motivated by real-world engineering applications, have been investigated to illustrate the accuracy, efficiency and applicability of the proposed method.

Keywords: Collapse load; Uncertain optimization; α-level strategy; Fuzzy safety assessment; Fuzzy limit analysis

1. Introduction

Classical limit analysis determines the ultimate loading capacities of engineering structures with ductile materials at plastic collapse. The excellent performance throughout previous engineering applications has forged classical limit analysis as a very reliable framework, which quantitatively assess the safety of engineering structures against the failure of plastic collapse. The success of classical limit analysis is attributed to the distinguished upper and lower bound (alternatively known as kinematic and static) theorems with certain modelling preconditions (i.e., perfect plasticity, sufficient ductility, geometry linearity etc.).

* Corresponding author. Tel.: +61 2 9385 4123; fax: +61 2 9385 6139.
E-mail address: di.wu@unsw.edu.au (D. Wu).

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Deterministic system parameters have always been utilized throughout classical limit analysis, which was believed to be adequate for traditional engineering purposes. However, the problem of uncertainties of system parameters has arisen along with the rapid increase of the complexity of modern engineering applications [1–6]. The mercurial nature, such as physical and geometric imperfections, manufacturing defections with human errors, seasonal variations with random environmental attacks, has imposed uncertainties upon system parameters. This inevitable phenomenon does influence the life-time performance of engineering structures [7–9]. Therefore, it is rational and indispensable to explicitly investigate the effect of uncertainties of system parameters of limit analysis upon ultimate structural strength against collapse [10–12].

Although fuzzy uncertainty analysis has been well established and advanced [13–16], there is relatively less focus on uncertain plastic limit analysis. In [17,18], fuzzy linear programs have been established and extended to structural plastic designs with uncertain design loading. In addition, a stochastic mathematical programming scheme has been implemented in [19–21] to tackle the stochastic limit and shakedown analysis. On the other hand, Kanno and Takewaki have modelled the uncertainty of applied loading by the non-stochastic Info-gap model in [22,23], which was firstly proposed by Ben-Haim [24,25], and the worst case of collapse load limits of truss structures were calculated by utilizing a mixed 0–1 programming approach. Furthermore, interval plastic limit analysis has been investigated in [26], in which the upper and lower bounds of the collapse load limit are determined by solving a pair of standard nonlinear programming problems.

Unlike aforementioned studies, this paper investigates the uncertain plastic limit analysis with fuzzy parameters which are simultaneously including the externally applied loads and yielding capacities of structural elements. A new computationally tractable structural safety assessment, namely the α-level collapse assessment (α-level CA) approach, is proposed to thoroughly investigate the fuzzy limit analysis of structures with rigid-perfectly plastic materials, such that the membership function of the collapse load limit of structure can be rigorously established. Essentially, the proposed α-level CA approach involves two components for constructing the membership function of the collapse load limit. The first step of this scheme utilizes the α-level strategy [27] to discretize the fuzzy limit analysis into a series of interval limit analyses. At each α-sublevel, the lower bound of the collapse load limit of interval limit analysis is obtained by solving a mathematical programming with equilibrium constraint (MPEC) problem mingled with the concept of the optimistic counterpart of uncertain optimization [28–30]. The MPEC is numerically tackled by a penalty algorithm which provides a robust framework for solving MPECs with remarkable performance [31,32]. On the other hand, the upper bound of the collapse load limit at each α-sublevel is solved by utilizing the concept of robust optimization [28–30], which takes the format as a standard linear programming (LP) problem. The advantage of this reformulation at each α-sublevel is that the quality of sharpness of the bounds is preserved with high computational efficiency. By successfully solving the upper and lower bounds of the collapse load limit at each α-sublevel, the membership function of the collapse load limit of structure can be established without any difficulties.

The paper is organized as follows. Section 2 firstly introduces the formulations of the deterministic limit analysis by using the mathematical programming approach. In the second part of Section 2, the uncertain limit analysis with fuzzy variables is formulated. Section 3 presents the proposed α-level CA approach for the construction of the membership function of the collapse load limit of engineering structure. The governing formulations of the determinations of the upper and lower bounds of the collapse load limit at each α-sublevel are also presented in Section 3. Three numerical examples, which are often encountered in engineering practice, have been assessed in Section 4 in order to illustrate the accuracy, applicability, as well as the computational efficiency of the proposed method. In the first numerical example, the membership function of the collapse load limit acquired from the α-level MP approach is validated by comparing with the Type I linear programming with interval coefficient (Type I LPIC) method (which is able to provide an analytical upper and lower bounds of interval linear programming problems within an exponential computational time [24]). In the second example, the computationally expensive Monte-Carlo simulation (MCS) method is employed to firstly verify the results, and secondly set off the robust performance of the α-level MP approach. The third example is motivated by real-life engineering application, which is meticulously selected to demonstrate the potential applicability and superior computational efficiency of the α-level CA approach. Finally, some concluding remarks are drawn in Section 5.