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Geometric Defuzzification revisited

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ABSTRACT

In this paper the *Geometric Defuzzification* strategy for type-2 fuzzy sets is reappraised. For both discretised and geometric fuzzy sets the techniques for type-1, interval type-2, and generalised type-2 defuzzification are presented in turn. In the type-2 case the accuracy of Geometric Defuzzification is assessed through a series of test runs on interval type-2 fuzzy sets, using *Exhaustive Defuzzification* as the benchmark method. These experiments demonstrate the Geometric Defuzzifier to be wildly inaccurate. The test sets take many shapes; they are not confined to those type-2 sets with rotational symmetry that have previously been acknowledged by the technique's developers to be problematic as regards accuracy. Type-2 Geometric Defuzzification is then examined theoretically. The defuzzification strategy is demonstrated to be built upon a fallacious application of the concept of *centroid*. This explains the markedly inaccurate experimental results. Thus the accuracy issues of type-2 Geometric Defuzzification are revealed to be inevitable, fundamental and significant.

1. Introduction

This paper is a critique of the technique of Geometric Defuzzification for type-2 fuzzy sets. Fuzzy set theory, and its correlate, fuzzy logic, were originated by Lotfi Zadeh [26] in the 1960s. Fuzzy logic is implemented in software through the Fuzzy Inferencing System (FIS), which may be of any type. The type of the FIS is determined by the highest type of the fuzzy sets employed within it. There are two sorts of type-2 FISs: 1. The Mamdani FIS for which the membership function output is a type-2 fuzzy set that requires defuzzification; 2. The Takagi-Sugeno-Kang FIS, with output membership functions that are either constant or linear; defuzzification is therefore unnecessary.

This paper, following the work on Geometric Defuzzification, is concerned solely with the Mamdani style FIS.

A Mamdani FIS (of any type) consists of three stages: fuzzification, inferencing, and defuzzification:

1. **Fuzzification** is the process in which a crisp input value's degree of membership of a fuzzy set is determined, based on the membership function of the fuzzy set.
2. **Inferencing,** which may be further subdivided into three stages: (a) Antecedent computation; (b) Implication; (c) Aggregation. The fuzzy set output by this stage is termed the *aggregated set*.
3. **Defuzzification**, in which the aggregated set is transformed into a crisp number, the output of the FIS.

The Geometric Defuzzification technique may be applied unproblematically to type-1 fuzzy sets [2].

Type-2 defuzzification conventionally breaks down into two stages [23] (Fig. 1):

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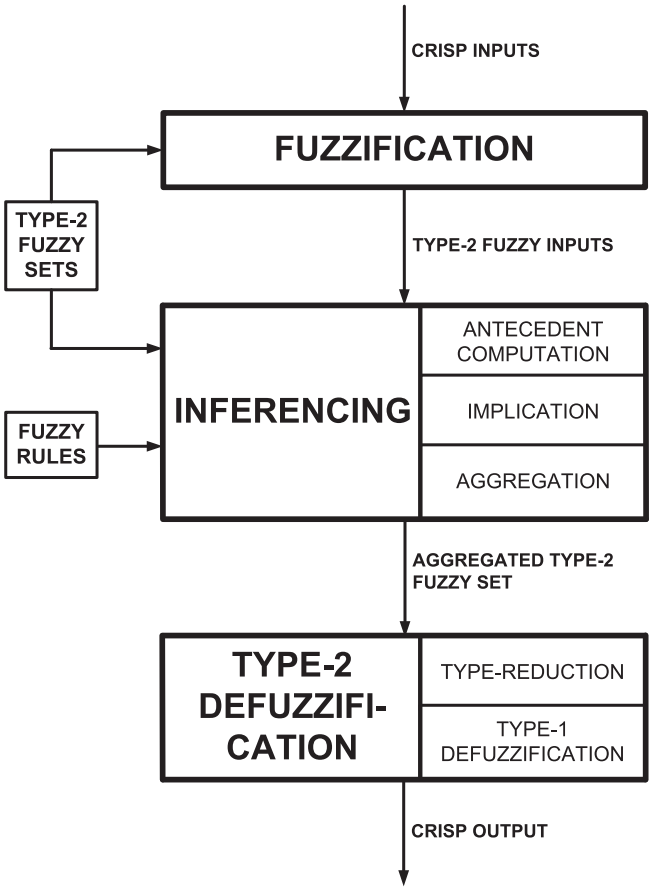

Fig. 1. The Mamdani Type-2 FIS [13]

1. **Type-reduction**, which transforms the type-2 fuzzy set into a type-1 fuzzy set;
2. **Type-1 defuzzification** of the resultant fuzzy set, converting it into a crisp number.

Type-reduction in the mathematically justified form of *Exhaustive Defuzzification* [11] (Section 3.1) is notoriously computationally complex, inspiring researchers to develop alternative approximate strategies [10–12,16–19], among them Coupland and John, the authors of the Geometric Defuzzifier. They express the motivation for their innovation thus [6, page 929]:

> "DEFUZZIFICATION is a critical stage in any Mamdani fuzzy inference system (FIS). It needs to be simple enough to permit system execution in real time on embedded hardware and yet must be precise and accurate so that correct decisions are taken."

The type-2 Geometric Defuzzifier is appealingly simple as it totally eliminates the stage of type-reduction. Dating from 2005, when the interval version was presented [4], the method has been published since in [6–8] in both its interval and generalised versions. In the context of the Geometric FIS, the technique has been presented in [2,3,5].

That this technique performs unreliably with certain forms of type-2 fuzzy set was acknowledged by its developers in 2008:

> "The only known form of a set that is problematic for the Geometric Defuzzifier is one that has rotational symmetry." [6, page 940]

This caveat was reiterated in 2013 [7, page 95].

The purposes of the present paper are:

- To show that the type-2 Geometric Defuzzifier's accuracy issues are pervasive and endemic across type-2 sets of *all* shapes, not just those that are rotationally symmetrical;
- To reveal the flawed reasoning underlying type-2 Geometric Defuzzification.

Geometric fuzzy sets are presented as differing conceptually from discretised fuzzy sets [2,5], but it is disputable whether the distinction is more than superficial. At any rate the definitions of geometric and discretised fuzzy sets permit creation