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Contradiction separation based dynamic multi-clause synergized automated deduction

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ABSTRACT

Resolution as a famous rule of inference has played a key role in automated reasoning for over five decades. A number of variants and refinements of resolution have been also studied, essentially, they are all based on binary resolution, that is, the cutting rule of the complementary pair while every deduction involves only two clauses. In the present work, we consider an extension of binary resolution rule, which is proposed as a novel contradiction separation based inference rule for automated deduction, targeted for dynamic and multiple (two or more) clauses handling in a synergized way, while binary resolution is its special case. This contradiction separation based dynamic multi-clause synergized automated deduction theory is then proved to be sound and complete. The development of this new extension is motivated not only by our view to show that such a new rule of inference can be generic, but also by our wish that this inference rule could provide a basis for more efficient automated deduction algorithms and systems.

1. Introduction

Resolution [34] as a famous rule of inference is particularly suitable for automation so has played a key role in automated reasoning for over five decades [12,31,37]. In developing resolution based automated deduction, dozens of variants and refinements of resolution have been studied from both the empirical and analytical sides aimed at improving the efficiency of the deduction process, for the detailed review and collection of different variations or strategies please refer to [6,8,12,13,15,37,40]. It is worthy note that those methods are all essentially based on binary resolution inference rule and indeed have improved the efficiency and capability of resolution based ATP systems in different ways.

In its simplest form, binary resolution may be viewed as a procedure for deducting a new clause (from the two 'source' clauses) which is a result of eliminating the occurrences of a complementary pair while leaving a disjunction of everything else. There are two key characteristics in the binary resolution: (1) it is based on the cutting rule of the complementary pair from two clauses respectively; (2) in the whole resolution deduction process every deduction involves only two clauses, so can be regarded as a static deduction process in term of the clause involvement.

This simple and elegant binary resolution inference scheme has been very successful, however there are still a lot of real problems unsolved or not solved efficiently as illustrated in TPTP (Thousands of Problems for Theorem Provers) [44].

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From the latest release of the TPTP benchmark library up to version 6.4.0 in 2016, there are 4,982 easy, 12,368 difficult, and 3,547 unsolved problems among 20,897 problems for theorem provers, where as indicated in [20], more than a third of these problems have more than 100 axioms, more than 10% have more than 1000 axioms, and more than 5% have more than 10,000 axioms. The efficiency and versatility of contemporary automated deduction depend on inference rule and techniques that may go beyond the pure resolution calculus, especially go beyond binary resolution [11,31].

This present work aims at addressing the following questions: although the simple and elegant binary resolution inference scheme has been successful, has it been too restrictive? Instead of treating a contradiction as a complementary pair based on two clauses, can we extend it into a contradiction consists of more than two clauses? Accordingly, can we make a flexible or dynamic selection of the number of clauses involved in each deduction to get better efficiency and capability?

There have been previous earlier attempts to use chains of rules (e.g., generalized resolution [13,36], hyper-resolution [35], and unit-resulting resolution [26]) which were not very successful. There has been up to now (to the best of our knowledge) no exist of theory and algorithm capable of handling multiple clauses dynamically in a synergized way, i.e., dynamic multi-clause synergized deduction.

Motivated by the above questions, plus our previous research work on resolution-based automated deduction based on many-valued logic [48], this paper proposes a new inference principle and its sound and complete automated deduction theory framework to extend from the existing static (i.e., fixed) binary resolution into a contradiction separation based dynamic multi-clause (two or more clauses) synergized inference rule. A key idea behind this new method is the extension of the concept of contradiction from a complementary pair based on two clauses to a typical kind of unsatisfiable clause set consists of more than two clauses. This typical kind of unsatisfiable clause set does not imply only one complementary pair among the clause set, therefore, the computation/searching is synergized among multiple clauses in terms of a contradiction.

The present work aims at establishing a new automated deduction theory with the following distinctive features in order to address the above questions: (1) *multi-clause deduction*: multiple clauses from a clause set (or even the whole clause set) are involved in each deduction process; (2) *dynamic and flexible deduction*: the number of clauses involved in each deduction can be varied from each other in the whole deduction process, so it is regarded as a dynamic and flexible deduction process; (3) *synergized deduction*: cooperative interaction among multiple clauses that creates a combined effect of all the clauses (two or more) on the deduction result, which can reflect better the overall logical relationship among multiple clauses than only considering two clauses several times; (4) *robust deduction*: deleting or adding some literals in the contradiction following certain strategies will not affect its contradictoriness as well as the corresponding deduction result; (5) *generic deduction*: it is generic, can be applied into a rich set of automated deduction systems, where the existing binary resolution rules and their variations are its special cases.

Note that an extended abstract of this paper was presented in [49]. The present paper provides a comprehensive introduction of all concepts and results with detailed proofs along with a good number of example illustrations. The focus of this paper is mainly on the new concept introduction and the corresponding automated deduction theory set up (i.e., soundness and completeness) to serve as a theoretical ground for the development of new provers. Therefore, the automated deduction theory is presented in a generic way, so that future work may easily build on it and explore various proof search strategies and implementation techniques. Actually, some specific algorithms and strategies to support this theory and achieve the implementation for automation with detailed experiments and case studies have been also established by the same author team, but are beyond the scope of this paper, so not covered here.

The remainder of this paper is structured as follows. In Section 2, we briefly review some related works. Followed some preliminaries about the notations and terminologies, the key concept of contradiction separation based deduction in propositional logic is provided in Section 3, along with soundness and completeness proved. Section 4 extends it into first-order logic. A graphical illustration of the key technical ideas is given in Section 5. The paper is concluded in Section 6.

2. Related work

As indicated in the Introduction section, a lot of variants of resolution or strategies have been studied from both the empirical and analytical sides. For example, strategies by restricting or specifying the resolution path [36,37], including set-of-support strategy as one of the most powerful strategies of this kind [47], semantic resolution [24,36], block resolution [24], linear resolution [32] and lock-resolution [4]; strategies like hyper-resolution to reduce the number of intermediate resolvents by combining several resolution steps into a single inference step [36]; strategies to specify the selection of clauses or literals [1,15,37,39]; resolution supplemented by heuristic strategies in the deduction process [7,10,16,38,40]; reducing the search space [9,25,29,33]; splitting the clause set [16]; reducing the function terms by using equality [28] etc., among others. These methods indeed improved the efficiency and capability of resolution deduction in different ways and in different extents, but they are all essentially based on binary resolution inference rule.

Some resolution deductions did consider to handle several clauses, such as Robinson [36] and Harrison and Rubin [13] independently proposed generalized resolution principle respectively, although both generalizations seem to handle several clauses, both share the key concept and have two essential features: (1) the clauses involved in the resolution are all binary clauses; (2) there must exist a special clause which includes a negation of literal appeared in those binary clauses. It is not easy to find that special clause in practical implementation. In addition, its soundness is still on the basis of binary resolution. Therefore, their works were not further developed and followed up since then. Another interesting one is hyper-resolution [36], which is a multi-step binary resolution process where intermediate clauses are discarded. The clauses