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On Convergence Properties of Implicit Self-paced Objective

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ABSTRACT

Self-paced learning (SPL) is a new methodology that simulates the learning principle of humans/animals to start learning easier aspects of a learning task, and then gradually take more complex examples into training. This new-coming learning regime has been empirically substantiated to be effective in various computer vision and pattern recognition tasks. Recently, it has been proved that the SPL regime has a close relationship with a implicit self-paced objective function. While this implicit objective could provide helpful interpretations to the effectiveness, especially the robustness, insights under the SPL paradigms, there are still no theoretical results to verify such relationship. To this issue, we provide some convergence results on the implicit objective of SPL. Specifically, we will prove that the learning process of SPL always converges to critical points of this implicit objective under some mild conditions. This result verifies the intrinsic relationship between SPL and this implicit objective, and makes the previous robustness analysis on SPL complete and theoretically rational.

1. Introduction

Self-paced learning (SPL) is a recently raised methodology designed through simulating the learning principle of humans/animals [7]. A variety of SPL realization schemes have been designed, and empirically substantiated to be effective in different tasks of computer vision and pattern recognition, such as object detector adaptation [16,19,21], specific-class segmentation learning [8], visual category discovery [9], concept learning [12], long-term tracking [3,20], graph trimming [25], co-saliency detection [27], matrix factorization [28,29], face identification [13], multi-task learning [11,17], multi-label learning [10], multimedia event detection [5,14] and so on.

To explain the underlying effectiveness mechanism inside SPL, [15] firstly provided some new theoretical understandings under the SPL scheme. Specifically, this work proved that the alternative optimization strategy (AOS) on SPL accords with a majorization minimization (MM) algorithm implemented on an implicit objective function. Furthermore, it is found that the loss function contained in this implicit objective has a similar configuration with non-convex regularized penalty (NCRP), leading to a rational interpretation to the robustness insight under SPL.

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However, albeit empirically verified in previous literatures [4,5], the previous theory still cannot completely substantiate the intrinsic convergence properties of SPL and its relationship with the implicit objective. The theory in [15] can only guarantee that during the iterations of SPL solving process (i.e., the MM algorithm), the implicit objective is monotonically decreasing, while cannot prove any convergence results on this implicit objective theoretically. However, theoretical results regarding this implicit objective are critical to the soundness of the robustness explanation of SPL, which guarantees to settle the convergence point of the algorithm down on the expected implicit objective, and intrinsically relate the original SPL model with this implicit objective.

To this theoretical issue of SPL, in this paper, we prove that the optimization of the implicit objective actually converges to critical points of original the SPL problem under satisfactorily weak conditions. This result provides an affirmative answer to our guess that the SPL intrinsically optimizes a robust implicit objective.

In what follows, we will first introduce some related background of this research in Section 2, and then we provide the main theoretical result of this work in Section 3. The detailed proofs can be found in Sections 4 and 6. Finally Section 7 is the conclusion.

2. Related work

In this section, we first briefly introduce the definition of SPL, and then provide its relationship with the implicit objective of NCRP.

2.1. The SPL objective

Given training data set $\{(x_i, y_i)\}_{i=1}^N$, many machine learning problems need to minimize an objective function of the following form:

$$J(w) = \phi_\lambda(w) + \sum_{i=1}^N L(y_i, g(x_i, w)),$$

where $w \in \mathbb{R}^D$ are variables to be solved, $\phi_\lambda(.)$ is a function representing the regularizer term of the objective, λ is a "paced" parameter enabling an "easy-to-hard" or "small-to-large"[2] learning procedure, $L(.)$ is the loss function and $g(\cdot, w)$ is the to-be-solved classifier or regression function. To improve the robustness, specially avoiding the negative influence brought by large-noise-outliers, SPL imposes additional importance weights $v = (v_1, \ldots, v_n)$ to loss functions of all samples, adjusted by a self-paced regularizer (SP-regularizer). Here, each $v_i \in [0, 1]$ represents how much weights should be imposed on the training samples (x_i, y_i) in the training process. The **self-paced objective** can then be designed as [5]:

$$E(w, v; \lambda) = \phi_\lambda(w) + \sum_{i=1}^N v_i L(y_i, g(x_i, w)) + f_\lambda(v_i), \tag{1}$$

where $f_\lambda(.)$ is the **SP-regularizer**, satisfying the following conditions:

1. $v \mapsto f_\lambda(v)$ is convex on [0,1];
2. Let
$$v_\lambda^*(l) = \arg\min_{v\in[0,1]} \{vl + f_\lambda(v)\},$$
then $l \mapsto v_\lambda^*(l)$ is non-increasing, and
$$\lim_{l\to 0} v_\lambda^*(l) = 1, \quad \lim_{l\to\infty} v_\lambda^*(l) = 0;$$
3. $\lambda \mapsto v_\lambda^*(l)$ is non-decreasing, and
$$\lim_{\lambda\to 0} v_\lambda^*(l) = 0, \quad \lim_{\lambda\to\infty} v_\lambda^*(l) \le 1.$$

Throughout this paper, we shall assume that $v_\lambda^*(l)$ can be uniquely determined and thus can be seen as a real-valued function instead of a set-valued function.[3]

The three conditions in the definition above provide basic principles for constructing a SP-regularizer. Condition 2 indicates that the model inclines to select easy samples (with smaller losses) in favor of complex samples (with larger

[2] Notice: "small-to-large" is through the weighting imposed on the losses of training examples.

[3] Under most of the current SP-regularizers, like the linear [9], mixture [28] and many others [1,10,15], $v_\lambda^*(l)$ can be uniquely induced for any value of l. However, in certain cases, it may occur that the optimum $v_\lambda^*(l)$ of (1) is not unique for certain SP-regularizer and certain l. The typical case is for the value of $l = \lambda$ under the hard SP-regularizer [7]. However, due to the condition (2) in the SP-regularization definition, i.e., $v_\lambda^*(l)$ is a non-increasing function with respect to l, actually the number of all such ls for any SP-regularizer is countable. Thus we still use such unique assumption for $v_\lambda^*(l)$ throughout paper since on one hand, it is of very low possibility to attain these points by our algorithm in real applications, and on the other hand such countable unexpected cases do not hamper the soundness of our proof.