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Adaptive decomposition-based evolutionary approach for multiobjective sparse reconstruction

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ABSTRACT

This paper aims at solving the sparse reconstruction (SR) problem via a multiobjective evolutionary algorithm. Existing multiobjective evolutionary algorithms for the SR problem have high computational complexity, especially in high-dimensional reconstruction scenarios. Furthermore, these algorithms focus on estimating the whole Pareto front rather than the knee region, thus leading to limited diversity of solutions in knee region and waste of computational effort. To tackle these issues, this paper proposes an adaptive decomposition-based evolutionary approach (ADEA) for the SR problem. Firstly, we employ the decomposition-based evolutionary paradigm to guarantee a high computational efficiency and diversity of solutions in the whole objective space. Then, we propose a two-stage iterative soft-thresholding (IST)-based local search operator to improve the convergence. Finally, we develop an adaptive decomposition-based environmental selection strategy, by which the decomposition in the knee region can be adjusted dynamically. This strategy enables to focus the selection effort on the knee region and achieves low computational complexity. Experimental results on simulated signals, benchmark signals and images demonstrate the superiority of ADEA in terms of reconstruction accuracy and computational efficiency, compared to five state-of-the-art algorithms.

1. Introduction

The sparse reconstruction (SR) problem widely exists in the under-determined system of linear equations [8,9,20], especially in the field of signal/image processing. There have been many successful applications in this field, such as action recognition [54], image super-resolution [22], image classification [12], human detection [41] and background subtraction [44]. In the SR problem, we want to recover the unknown sparse signal $\mathbf{x}$ from the measurement $\mathbf{b}$ in the following under-determined system

$$\mathbf{b} = \mathbf{A}\mathbf{x} \tag{1}$$

where $\mathbf{x}$ is an unknown sparse vector with k nonzero elements ($\mathbf{x} \in \Re^n$, $k \ll n$), $\mathbf{A}$ is a full-rank sensing matrix ($\mathbf{A} \in \Re^{m \times n}$, $m < n$) and should satisfy the restricted isometry property (RIP) [8], and $\mathbf{b}$ is the measurement vector.Sometimes the signal

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$\mathbf{x}$ is not sparse but has a sparse representation $\mathbf{w}$ with respect to some bases $\boldsymbol{\Psi}$, such as the Fourier bases and the wavelet bases. In this case, Eq. (1) can be transformed to $\mathbf{b} = \mathbf{A}\boldsymbol{\Psi}\mathbf{w}$, and we only need to reconstruct $\mathbf{w}$ before acquiring $\mathbf{x}$ by $\mathbf{x} = \boldsymbol{\Psi}\mathbf{w}$. In this paper, matrices are denoted by bold upper case letters and vectors are highlighted by boldface lower-case letters.

For simplicity, here we focus on the case that the signal $\mathbf{x}$ itself is sparse. The compressed sensing theory [10] can be employed to reconstruct $\mathbf{x}$ by solving the following SR problem:

$$\min_{\mathbf{x}} \ \|\mathbf{x}\|_0, \ s.t. \ \mathbf{A}\mathbf{x} = \mathbf{b} \tag{2}$$

where $\|\mathbf{x}\|_0$ represents the number of nonzero elements in $\mathbf{x}$, and $\|\cdot\|$ denotes the standard Euclidean norm for a vector. If noise is included in the measurements, this problem is updated to

$$\min_{\mathbf{x}} \ \|\mathbf{x}\|_0, \ s.t. \ \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2 \leqslant \sigma \tag{3}$$

where $\sigma > 0$ is a given constant related to the noise.

The problem in (2) is known to be NP-hard [36]. Quite a few algorithms have been developed to solve this l_0-norm SR problem, such as orthogonal matching pursuit (OMP) [7,39], compressive sampling matching pursuit [37] and iterative hard thresholding methods [5,45]. They perform well only when the measurement samples are much more than nonzero elements in $\mathbf{x}$. The l_0-norm problem can also be transformed to a convex optimization problem [11], or relaxed by the l_1-norm [4,19,42,46] or $l_p(0 < p < 1)$-norm [33,47,49,52]. The l_1-norm algorithms are more robust to noise and can recover signals with better reconstruction quality. But under some cases (e.g., the matrix $\mathbf{A}$ does not satisfy the low coherence conditions), they can hardly guarantee the equivalence between l_1-norm and l_0-norm. The $l_p(0 < p < 1)$-norm is nonconvex and nonsmooth. Its convergence is yet to be proven theoretically, and it is very challenging to derive fast and efficient solutions for $l_p(0 < p < 1)$-norm problems. For these relaxation algorithms, the problem in (2) is commonly transformed to the following continuous optimization problem

$$\mathbf{x} = F(\mathbf{x}) = arg\min_{\mathbf{x}} \ \lambda\|\mathbf{x}\|_p + \frac{1}{2}\|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2 \tag{4}$$

where $p \in (0, 1]$ and λ denotes the regularization parameter. Here exists the problem of determining λ to balance the objective term and the penalty function term$\|\mathbf{x}\|_1$, as λ is closely related to the reconstruction quality. However, there is no exact method for finding the optimal λ value in practical applications.

All of the above algorithms we mentioned so far are single objective, and they solve the combined objective function in an independent way, where the solution path is fixed. To exploit joint optimization and provide adaptability to the solution path, a new approach is proposed to transform Problem (4) to a multiobjective sparse reconstruction (MOSR) problem:

$$f(\mathbf{x}) = \min_{\mathbf{x}} \left(\|\mathbf{x}\|_0, \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2\right) \tag{5}$$

where $\|\mathbf{x}\|_0$ and $\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2$ represent sparsity and measurement error respectively.

Multiobjective evolutionary algorithms (MOEAs), such as NSGA-II [17], differential evolution [38] and MOEA/D [53], are widely used to tackle optimization problems with two or more objectives. MOEAs optimize all the objectives simultaneously and can provide a variety of trade-off solutions (termed as Pareto front, PF) among the objectives. Recently, MOEAs, such as the soft-thresholding evolutionary multiobjective (StEMO) algorithm [32], are applied to solve the MOSR problem. StEMO is based on the NSGA-II framework and is incorporated with the soft-thresholding algorithm [26] for local search to further improve the convergence performance. The knee region on the final PF is proved to provide the best trade-off, because it has the largest marginal rates of return [32]. It is selected as the final solution and can be identified by the angle-based method [6]. Another two algorithms, an improved MOEA/D with L1/2 solver [30] and sparse preference based local search [31], abbreviated as MOEA/D-L1/2 and SPLS respectively, both integrate iterative threshold algorithms into MOEAs and find a local part of Pareto front near the knee region with preference. They use a single starting solution from chain search results and weakly Pareto front respectively for local search, and then execute multiple truncations to update the solution set and increase the diversity of population. In [48], the LBEA algorithm is developed by embedding a linear Bregman-based [27] local search operator into the differential evolution paradigm. An adaptive strategy is designed for the linear Bregman-based local search, where the number of individuals and iterations for local search is adaptive to accelerate convergence.

These algorithms demonstrate the advantages of MOEA in solving the MOSR problem. However, they can not provide fast reconstruction speed due to the use of the Pareto nondominance principle. This can be verified by a simple experiment. Assuming a sparse signal with length $N = 1000$ and sparsity ratio $\frac{k}{n} = 0.05$. The nonzero elements are randomly chosen from the standard normal distribution. The sensing matrix is a Gaussian random matrix with the dimension 400×1000. The population size and maximum number of generation are both set to 100. Take StEMO and LBEA as examples, the average running time is shown in Fig. 1. We can see that the Pareto dominance-based selection operators consume the most time in both algorithms.

On the other hand, the reconstruction quality of these algorithms is limited. For StEMO and LBEA, they put search effort uniformly over the whole PF. However, the knee region has the solutions with the maximum marginal rates of return, which deserves more search effort. Even if in cases when the knee region does not provide the best approximation for the ground-truth data, the solutions in this region are still Pareto optimal [32]. Therefore, the computational effort of StEMO