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A goal programming approach to deriving interval weights in analytic form from interval Fuzzy preference relations based on multiplicative consistency

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ABSTRACT

This paper focuses on how to find an analytic solution of optimal interval weights from consistent interval fuzzy preference relations (IFPRs) and obtain approximate-solution-based interval weights in analytic form from inconsistent IFPRs. The paper first analyzes the popularly used interval weight additive normalization model and illustrates its drawbacks on the existence and uniqueness for characterizing]0, 1[-valued interval weights obtained from IFPRs. By examining equivalency of]0, 1[-valued interval weight vectors, a novel framework of multiplicatively normalized interval fuzzy weights (MNIFWs) is then proposed and used to define multiplicatively consistent IFPRs. The paper presents significant properties for multiplicatively consistent IFPRs and their associated MNIFWs. These properties are subsequently used to establish two goal programming (GP) models for obtaining optimal MNIFWs from consistent IFPRs. By the Lagrangian multiplier method, analytic solutions of the two GP models are found for consistent IFPRs. The paper further devises a two-step procedure for deriving approximate-solution-based MNIFWs in analytic form from inconsistent IFPRs. Two visualized computation formulas are developed to determine the left and right bounds of approximate-solution-based MNIFWs of any IFPR. The paper shows that this approximate solution is an optimal solution if an IFPR is multiplicatively consistent. Three numerical examples including three IFPRs and comparative analyses are offered to demonstrate rationality and validity of the developed model.

1. Introduction

Decision problems with multiple criteria are often faced in daily lives, business situations and industrial engineering. To make a reasonable decision, a number of multi-criteria decision making (MCDM) methods have been developed over the past few decades [45]. The traditional analytic hierarchy process (AHP) [22] as a common MCDM method uses the paired comparison framework to elicit decision-makers' judgments under a bipolar scale with a ratio-based neutral value of 1, and these judgments are constituted as multiplicative reciprocal comparison matrices (also called multiplicative preference relations [33]). Another popular bipolar scale [8] is the unit interval whose neutral value is set to be 0.5. The paired comparison results under the unit interval scale are constituted as additive reciprocal comparison matrices [5] (also called fuzzy preference relations (FPRs) [19]). Because of the complex information granularity [20,21] and increasing complexity of MCDM problems, the paired comparison results often have ambiguity and indeterminacy [7,9]. As a consequence, interval

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multiplicative preference relations [23] and interval fuzzy preference relations (IFPRs) [42] have been introduced and applied in solving real-life MCDM problems [12,13,17,18,26,27,30,38–40].

The main advantage of the paired comparison method is that a decision maker can focus exclusively on two compared objects at a time, which facilitates the elicitation of subjective judgments [3,22]. However, this way often limits the decision maker's global consciousness for all compared objects, implying that the subjective preferences in a preference relation may involve inconsistent judgments [6,15,41]. In addition, an unsuitable priority weight vector may be obtained from a preference relation with high inconsistency [3,11]. Therefore, it is important to model consistency of subjective preferences for obtaining appropriate priority weights from preference relations.

Multiplicative consistency models of IFPRs in the current research can be classified into two groups. One is the feasible region based models [16,43], which use the existence of a consistent FPR within an IFPR to determine consistency of IFPRs. The other group is the mathematical constraint based models [14,29,31,32,34]. The requirement of the feasible region based consistency models is very weak due to the fact that a number of consistent FPRs and inconsistent FPRs may be within an IFPR. On the other hand, as was already pointed out by Krejčí [14], the mathematical constraint based consistency models [31,34] highly depend on the numbering of compared objects and the consistency models [29,32] do not preserve additive reciprocity of paired comparisons. Based on the functional transitivity equation by Chiclana et al. [4], Krejčí [14] used the constrained interval arithmetic to define multiplicatively consistent IFPRs. This consistency definition can maintain two important properties: invariance with respect to permutation of compared object numberings [1] and additive reciprocity of paired comparisons [10]. However, it is hard to use the consistency model [14] to find an analytic solution to optimal interval weights of consistent IFPRs (see a further analysis in Example 4). These issues lead to a research motivation on developing a new framework for modelling multiplicative consistency of IFPRs.

Deriving suitable interval weights from IFPRs plays a crucial role for MCDM with IFPRs. Different interval weight generation methods have been proposed in the literature. For instance, Genç et al. [11] and Xia and Xu [34] devised max-min based approximate computation formulas. Some researchers developed different optimization models to obtain interval weights from IFPRs, such as the linear programming model [43], goal programming (GP) models [29,44], the logarithmic least square model [28] and the nonlinear programming model [35]. However, the obtained solution by the existing interval weight generation methods is often not the most suitable solution for multiplicatively consistent IFPRs (see Example 2 in Section 6). This is mainly due to two facts: (1) These methods are based on defective consistency models mentioned before; (2) The popularly used interval weight additive normalization condition [24] is employed as a constraint of the existing optimization models while this additive normalization model has deficiencies on the existence and uniqueness for characterizing]0, 1[-valued interval weight vectors (see a further examination in Section 3.1).

To overcome aforesaid defects, this paper introduces equivalency of]0, 1[-valued interval weight vectors from the viewpoint of generating IFPRs and proposes a new normalization condition to define multiplicatively normalized interval fuzzy weights (MNIFWs). A transformation method is put forward to convert a]0, 1[-valued interval weight vector into an equivalent MNIFW vector, and a likelihood formula is devised to compare and rank MNIFWs. The paper uses MNIFWs to define multiplicatively consistent IFPRs, and presents significant properties for indeterminacy indices of multiplicatively consistent IFPRs and their associated MNIFWs. Based on these properties, two GP models are developed to find an analytic solution of optimal MNIFWs of consistent IFPRs. The two GP models are equivalently converted into two least square models and their analytic solutions are found by the Lagrangian multiplier method. Based on the analytic solutions, optimal MNIFWs in analytic form are derived from multiplicatively consistent IFPRs. Subsequently, a two-step procedure is established to find approximate-solution-based MNIFWs in analytic form from inconsistent IFPRs. The paper develops two visualized computation formulas used to determine the left and right bounds of approximate-solution-based MNIFWs of any IFPR.

The remainder of this paper is structured as follows. Section 2 provides preparations on multiplicatively consistent FPR, IFPRs and the geometric mean based indeterminacy index of an IFPR. Section 3 illustrates deficiencies of the interval weight additive normalization model and introduces a novel framework of MNIFWs. Section 4 defines multiplicatively consistent IFPRs and proposes their properties. GP models are developed to derive optimal or approximate-solution-based MNIFWs in analytic form from consistent or inconsistent IFPRs in Section 5. Section 6 offers three numerical examples including comparative analyses to illustrate the developed model. Finally, conclusions are covered in Section 7.

2. Preliminaries

Let $X = \{x_1, x_2, ..., x_n\}$ be a non-empty set of objects (e.g. alternatives or criteria) and $R = (r_{ij})_{n\times n}$ be a matrix, then R is called an FPR on X if R satisfies

$$0 \le r_{ij} \le 1, r_{ij} + r_{ji} = 1, r_{ii} = 0.5, i, j = 1, 2, ..., n, \tag{2.1}$$

where r_{ij} indicates a [0, 1]-valued preference of object x_i over x_j.

Obviously, the stronger the preference r_{ij}, the greater the ratio $\frac{r_{ij}}{r_{ji}} = \frac{r_{ij}}{1-r_{ij}}$ if $0 < r_{ij} < 1$. Moreover, if $1 > r_{ij} > 0.5$, we have $\frac{r_{ij}}{r_{ji}} > 1$, implying that object x_i is preferred to x_j having a ratio $\frac{r_{ij}}{r_{ji}}$. If $0 < r_{ij} < 0.5$, then we have $\frac{r_{ij}}{r_{ji}} < 1$, indicating that x_i is non-preferred to x_j having a ratio $\frac{r_{ij}}{r_{ji}}$. In particular, if $r_{ij} = 0.5$, then $\frac{r_{ij}}{r_{ji}} = 1$, meaning that x_i and x_j are indifferent.