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Stochastic self-triggered MPC for linear constrained systems under additive uncertainty and chance constraints☆

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ABSTRACT

This paper presents a stochastic self-triggered model predictive control (MPC) scheme for linear systems with additive uncertainty, and with the states and inputs being subject to chance constraints. In the proposed control scheme, the succeeding sampling time instant and current control inputs are computed online by solving a formulated optimization problem. The chance constraints are reformulated into a deterministic fashion by leveraging the Cantelli's inequality. Under few mild assumptions, the online computational complexity of the proposed control scheme is similar to that of a nominal self-triggered MPC. Furthermore, initial constraints are incorporated into the MPC problem to guarantee the recursive feasibility of the scheme, and the stability conditions of the system have been developed. Finally, numerical examples are provided to illustrate the achievable performance of the proposed control strategy.

1. Introduction

Stochastic model predictive control (MPC) has received a considerable attention because it is capable of optimizing the system performance under stochastic uncertainties and chance constraints on the state and input variables. The development of stochastic MPC has stimulated a wide range of applications in industry, such as building climate control [28,31] and automotive control [4,21]. In contrast to robust MPC which relies on the worst-case consideration on the uncertainties, stochastic MPC makes use of the information about the distribution of the uncertainties. If the uncertainties are characterized as random processes, it is desirable to reformulate the constraints in a probabilistic framework. Also, stochastic MPC caters for many cases in which the constraints are probabilistic in nature. As shown in [27], for the same prediction horizon, if the constraints are formulated as chance constraints, the region of attraction will be enlarged significantly.

Two cruxes exist in the design of a stochastic MPC algorithm:: (i) reformulating chance constraints into deterministic representations, and (ii) theoretically analyzing stability and recursive feasibility. As stated in [11] two main approaches to the former have been proposed to handle the optimization problem with chance constraints: analytical approximation methods and scenario-based methods. For linear systems subject to additive uncertainties, various methods cast the stochastic optimal control problems with chance constraints as a tractable problem by deterministic reformulations of the probabilistic constraints [6,7,18,26]. The online computational complexity of the resulting algorithm is comparable to that of a nominal

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MPC but some degree of conservativeness is introduced due to the approximation of the chance constraints. Alternatively, in scenario-based methods [5,27,34], a set of disturbance realizations is randomly generated to find the optimal solution of the stochastic MPC problem with an arbitrarily high accuracy. It is worth noting that scenario-based methods cope with generic probability distributions, cost functions and chance constraints. In comparison with analytical methods, the resulting algorithm is computationally demanding because a large number of disturbance realizations is required for the on-line computation.

As discussed above, the stochastic MPC schemes are executed periodically on digital platforms. In networked control systems (NCSs), whose components are connected through a communication network, the communication cost among components cannot be neglected, and the high communication load is a main concern for implementing stochastic MPC. If the components are connected through wireless networks, the communication load will be heavier, possibly leading to packet dropouts and network-induced delays. These challenges, introduced by the communication network, may degrade the system performance and even destabilize the control system [13,38]. To deal with these challenges, the aperiodic sampling scheme is a promising solution since a considerable amount of communication load can be reduced. For consensus problems in the multi-agent system in which agents share information through the networks, a novel event-triggered transmission strategy is reported in [15]. Reviews on NCSs considering the aperiodical control and filtering schemes are referred to [9,20,39,40].

In periodic sampling schemes, without considering the particular dynamics of the system, this general implementation can lead to redundant samplings. However, in aperiodic sampling schemes, the control inputs are updated only when the system performance cannot meet some specified requirements (i.e., the performance index violates some predefined thresholds), and this sampling mechanism can lead to a lower average sampling rate. Results in [1,17] have highlighted these advantages, and since then, several results on MPC using aperiodic sampling schemes have been proposed. Recently, robust event-triggered MPC and robust self-triggered MPC have been proposed, and these control strategies have received increasing attention. For nonlinear continuous-time systems affected by additive uncertainties, an event-triggered robust MPC algorithm has been proposed in [19]. For nonlinear input-affine dynamical systems, a self-triggered MPC control scheme, in which the control sequence is adaptively sampled is reported in [16]. For linear systems, the co-design problem of jointly determining the control input to the plant and the next sampling instant has been discussed in [8,14,22]. The authors in [2,3] separate the problem to a bilevel optimization problem while tube-based MPC is utilized to deal with the additive disturbance.

Note that the aforementioned self-triggered MPC algorithms only consider hard constraints, with an exception in a most recent work [8], where the probabilistic constraints are considered. In this study, we aim to develop a stochastic MPC algorithm for disturbed linear systems under the framework of *self-triggered mechanism*. The difference between our proposed work and the existing work is that we take the possibly unbounded stochastic uncertainty and chance constraints into account. The difference between SMPC and RMPC makes our work essentially different from the existing work on robust self-triggered MPC. As a result, the main challenges of this work are: How to propagate the uncertainties during two sampling instants; and how to formulate a tractable optimization problem in the presence of chance constraints. Comparing with [8], chance constraints in our work are reformulated in a completely different way; consequently, the resulting theoretical analysis is inherently different. The main contribution of this work is two-fold:

- A stochastic self-triggered MPC scheme is proposed for linear systems under additive uncertainty and chance constraints. The chance constraints on the states and inputs are reformulated into deterministic terms by leveraging the Cantelli's inequality [10,12]. At each sampling time instant, the co-design problem of deciding the next sampling instant and the control input sequence during the inter-execution interval is addressed by solving a set of optimization problems. With the proposed aperiodic scheduling strategy, the controller only needs to sample the state and transmit control input when necessary, therefore reducing the communication load between the sensor and controller significantly.
- Theoretical analysis of the proposed stochastic self-triggered MPC algorithm is performed. Tightened constraints on the state and control input are designed to guarantee the satisfaction of chance constraints of the proposed control scheme. In [8], the recursive feasibility is guaranteed based on the assumption that the uncertainty is bounded while in our method, additional initial constraints are imposed to ensure the recursive feasibility of the scheme. Meanwhile, sufficient conditions under which the closed-loop system is stable are given, and it has been shown that the system state will converge to an invariant set around the origin.

The remainder of this paper is organized as follows. Section 2 introduces the formal problem formulation of the work, where the reformulation of chance constraints and constructions of constraint sets are presented. In Section 3, the proposed stochastic self-triggered MPC problem is defined. Following that, the closed-loop properties of the proposed control scheme are summarized in Section 4, and sufficient conditions to guarantee the stability of the system are given. In Section 5, the advantages of the proposed control scheme are demonstrated by numerical examples. Section 6 concludes the paper.

Notations: $\mathbb{N}$ denotes the set of integers, and $\mathbb{N}_{[a,b]}$ represents the set of integers from a to b, where $a \le b, a, b \in \mathbb{N}$. $\mathbb{R}^n$ stands for the n-dimensional real space. $\mathbb{E}\{\cdot\}$ and $\text{var}\{\cdot\}$ denote the expectation and variance of a random variable, respectively. For a matrix X, X^{T} denotes the transpose of X, and $\text{tr}(X)$ denotes the trace of X. $X = \text{diag}(x_1, x_2, \dots, x_n)$ denotes a diagonal matrix with elements $x_1, x_2, \dots, x_n$. The maximum and minimum eigenvalues of X are denoted by $\bar{\lambda}(X)$ and $\underline{\lambda}(X)$, respectively. Given a set $\mathcal{B}$ and a point η, $d(\eta, \mathcal{B}) := \inf\{\|\eta - b\|, b \in \mathcal{B}\}$ denotes the point-to-set distance. $\mathcal{B}_r := \{x \in \mathbb{R}^n : \|x\| \le r\}$ denotes the ball with a radius of r around the origin.