Information Sciences 000 (2017) 1–7

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Intuitionistic fuzzy interpretations of Barcan formulas☆

Krassimir Atanassov[a,b]

[a] Department of Bioinformatics and Mathematical Modelling, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences Acad. G. Bonchev Str., Bl. 105, Sofia-1113, Bulgaria
[b] Intelligent Systems Laboratory, Prof. Asen Zlatarov University, Bourgas-8000, Bulgaria

A R T I C L E I N F O

Article history:
Received 6 April 2017
Revised 26 October 2017
Accepted 28 October 2017
Available online xxx

MSC:
03E72

Keywords:
Barcan formulas
Intuitionistic fuzzy implication
Intuitionistic fuzzy logic
Intuitionistic fuzzy quantifier

A B S T R A C T

In a series of papers, 186 different intuitionistic fuzzy implications have been defined. For each of them, the four Barcan formulas are checked in the case of the standard intuitionistic fuzzy quantifiers. The list of the intuitionistic fuzzy implications that satisfy the Barcan formulas is given. Modifications of these formulas for the case of the extended intuitionistic fuzzy quantifiers are introduced and an open problem for them is formulated.

1. Introduction

Intuitionistic fuzzy propositional calculus, intuitionistic fuzzy predicate logic, intuitionistic fuzzy modal logic and intuitionistic fuzzy temporal logic have been developed since the mid 1980s. In a lot of papers, different results related to these logics were published. The most important of them were collected in the book [5].

To each proposition (sentence) of classical logic (e.g., [10]), we juxtapose its truth value: truth – denoted by 1, or falsity – denoted by 0. In fuzzy logic [11], the truth value, called “truth degree”, is a real number in the interval [0, 1]. In the intuitionistic fuzzy case (see [1,5]), one more value – “falsity degree” – is added. It is again in the interval [0, 1]. Thus, to the proposition p, two real numbers, $\mu(p)$ and $\nu(p)$, are assigned with the following constraint:

$$\mu(p) + \nu(p) \leq 1.$$

Let $\mathcal{S}$ be a set of propositions and let us have the evaluation function V: $S \to [0, 1] \times [0, 1]$, in such a way that: $V(T) = \langle 1, 0\rangle$, $V(F) = \langle 0, 1\rangle$, where T and F are the logical truth and falsity, respectively, and

$$V(p) = \langle \mu(p), \nu(p)\rangle.$$

Many conjunctions, disjunctions, negations and implications have been defined (see [5]). Currently, there are 186 different intuitionistic fuzzy (IF) implications. The first 185 of them were collected in [5] and the 186th was published in [7].

In [2,4,5], other eight quantifiers were introduced and some of their properties were studied.

☆ To Janusz on the occasion of his 2 × 35 years!
E-mail address: krat@bas.bg

https://doi.org/10.1016/j.ins.2017.10.052

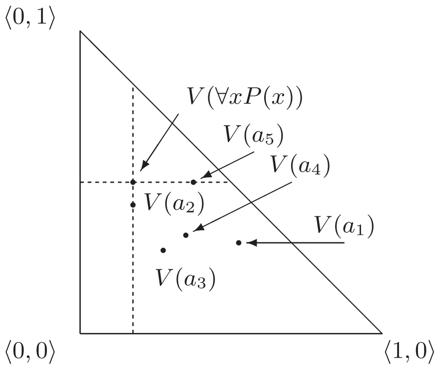

Fig. 1. Geometrical interpretation of quantifier ∀.

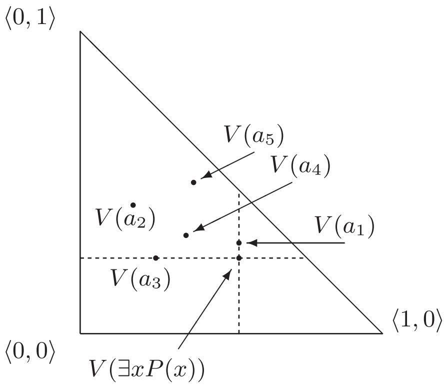

Fig. 2. Geometrical interpretation of quantifier ∃.

Let $x \in E$ be a variable and $P(x)$ be a predicate. Let

$$V(P(x)) = \langle \mu(P(x)), \nu(P(x)) \rangle.$$

In [2], the IF interpretations were introduced of the quantifiers *for all* (∀) and *there exists* (∃) by

$$V(\exists x P(x)) = \langle \sup_{z \in E} \mu(P(z)), \inf_{z \in E} \nu(P(z)) \rangle,$$

$$V(\forall x P(x)) = \langle \inf_{z \in E} \mu(P(z)), \sup_{z \in E} \nu(P(z)) \rangle.$$

For the finite set E, we can use the denotations

$$V(\exists x P(x)) = \langle \max_{z \in E} \mu(P(z)), \min_{z \in E} \nu(P(z)) \rangle,$$

$$V(\forall x P(x)) = \langle \min_{z \in E} \mu(P(z)), \max_{z \in E} \nu(P(z)) \rangle.$$

Below, we use only the first forms of both quantifiers and we name them “standard intuitionistic fuzzy quantifiers”.

Their geometrical interpretations are illustrated in Figs. 1 and 2, respectively, where $a_1, \ldots, a_5$ are the possible values of variable x and $V(a_1), \ldots, V(a_5)$ are their IF evaluations.

It is important to mention that the two quantifiers juxtapose to predicate P a point (exactly one per quantifier) in the IF interpretational triangle.

In [4,5], the following six new quantifiers were introduced and named “extended intuitionistic fuzzy quantifiers”. There, some of their properties were studied.

$$V(\forall_\mu x P(x)) = \{\langle x, \inf_{z \in E} \mu(P(z)), \nu(P(x)) \rangle | x \in E\},$$

$$V(\forall_\nu x P(x)) = \{\langle x, \min(1 - \sup_{z \in E} \nu(P(z)), \mu(P(x))), \sup_{z \in E} \nu(P(z)) \rangle | x \in E\},$$