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# Synchronization regions of discrete-time dynamical networks with impulsive couplings☆

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ABSTRACT

This paper deals with synchronization of a class of discrete-time dynamical networks. First, a novel model for discrete-time dynamical networks with impulsive couplings between nodes is proposed. Second, both global and local stability of synchronization manifold is investigated using the multiple Lyapunov function method and some inequality techniques. As a result, several synchronization criteria are obtained. Third, two illustrative examples are given to validate the effectiveness of the proposed synchronization criteria. Analysis on the synchronization regions of the discrete-time dynamical network with metapopulation dynamics is further made, which reveals such a finding that either a relatively large coupling strength or a short impulsive interval is not necessarily beneficial to synchronization of the discrete-time dynamical network.



## 1. Introduction

Recent years have witnessed increasing interests in studying synchronization of complex dynamical networks due to the fact that synchronization is closely related to various research topics in practical engineering control systems, such as power grids, sensor networks, and cellular neural networks [3,7,8,30,34,40]. During the past decade, a great number of notable results have been reported on synchronization control and stability analysis of complex dynamical networks, see, e.g. [6,13,24,25,38].

Recalling some existing results, it is found that several control methods have been applied to solve the synchronization problem of complex dynamical networks, for example, adaptive control, sampled-data control, impulsive control and sliding mode control [41,43], and so on. Specifically, the impulsive control method has gained much attention and become an important control approach. The main reason is that an impulsive control strategy can save network bandwidth

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effectively, and thereby reduce communication or control costs of networked control systems. In fact, impulses in the impulsive control are essentially samples of system states at a sequence of discrete instants [4], and only sampled data at a few discrete instants are used to stabilize a given network. Synchronization of complex dynamical networks based on an impulsive control strategy has been investigated in a number of publications. To mention a few, synchronization of impulsively coupled continuous-time dynamical networks is studied in [15,26,37,45], where the couplings between nodes occur at discrete instants; Synchronization of complex dynamical networks with hybrid couplings is investigated via aperiodically intermittent control [26,35]; Synchronization of two nonidentical complex dynamical networks via periodically intermittent pinning control is considered in [39]; Synchronization of multi-agent systems [12,49] using an event-triggered scheme [42,47] is discussed in [5,11,13]; Multi-consensus of multi-agent systems with various intelligences using switched impulsive protocols is taken into account in [14]; and synchronization of dynamical networks or multi-agent systems via distributed impulsive control is analyzed using linear matrix inequality (LMI) techniques [16–18].

A common feature of above results is that the complex dynamical networks under study are focused on continuous-time models rather than discrete-time models. It should be pointed out that, for discrete-time dynamical networks [9,21,27,29,31], few results have been reported on synchronization using impulsive control strategies. The main difficulty lies in how to stabilize a dynamical network based on control signals only at a few discrete instants, especially when nonlinearity (such as chaotic nature in self-dynamics of nodes) and information constraints (such as time delays and packet loss [12,44,46]) exist in the dynamical network. To overcome this difficulty, the multiple Lyapunov function method, together with some inequality techniques including LMIs, is usually employed, and a number of important results have been derived. For instance, in [4], synchronization is investigated for two identical discrete-time delayed neural networks by linear-state feedback impulsive controllers; Impulsive perturbations on the stability of discrete-time complex-valued neural networks is studied in [33]; Impulsive effects on the synchronization of discrete-time complex networks with stochastic noise is analyzed in [22]; and impulsive synchronization of discrete-time networked oscillators with partial input saturation is also investigated in [23]. However, when impulsive couplings exist in a discrete-time dynamical network, few results have been reported on the synchronization issue, which motivates the current study.

In this paper, we deal with synchronization of discrete-time dynamical networks by introducing a discrete-time dynamical network model, in which couplings between nodes occur *impulsively* at discrete time instants. This model is motivated by a few natural and man-made networked systems. For example, animal groups interact and mate *seasonally*, which influences their metapopulation dynamics; Ants exchange and transfer food information *only at some instants*, by which their social collaboration is rather efficient; In distributed sensor networks, sensors broadcast their collected information to neighbors *only periodically* so that an economical electrical energy consumption can be maintained. The main contributions of this paper can be summarized as follows

- A novel model of discrete-time dynamical networks with impulsive couplings is proposed, which is inspired by some real-world phenomena and application scenarios as well as some existing results on continuous-time dynamical networks [15,26,37,45];
- by employing a multiple Lyapunov function method and inequality techniques, both global and local stability analysis of the synchronization manifold is made for the proposed network model, and two synchronization criteria are obtained. It is revealed that synchronization is determined by network topology, coupling strength, and impulsive intervals; and
- an application of the obtained theoretical results to metapopulation dynamics is presented, which shows how self-dynamics of nodes and impulsive coupling strengths produce diverse synchronization regions of the metapopulation network. Moreover, synchronization regions of the discrete-time dynamical network with different settings of system parameters are also investigated in depth. It is worth mentioning that the obtained synchronization criteria are of a simple form, which enables us to analyze complex relations between model parameters and to study the effects of impulsive couplings by visualizing synchronization regions when specific self-dynamics are given.

This paper is organized as follows. Section 2 gives some preliminaries and defines our discrete-time dynamical network model with impulsive couplings. Synchronization criteria for global and local stability are then explored for the discrete-time dynamical networks in Section 3.1 and Section 3.2, respectively. Section 4 uses two illustrative examples to validate the effectiveness of the obtained synchronization criteria. In Section 5, synchronization regions of the discrete-time dynamical networks with impulsive couplings are analyzed according to different characteristics of individual dynamics. Finally, concluding remarks are made in Section 6.

Throughout the paper, $||x||$ represents the Euclidean norm $||\cdot||_2$ for a vector $x \in R^n$; and $||A||$ denotes the 2-norm for a matrix $A \in R^{N \times N}$, i.e., $||A|| = [\lambda_{max}(A^T A)]^{1/2}$.