Surrogate-assisted hierarchical particle swarm optimization

Haibo Yu\textsuperscript{a}, Ying Tan\textsuperscript{b}, Jianchao Zeng\textsuperscript{a,c}, Chaoli Sun\textsuperscript{b,*}, Yaochu Jin\textsuperscript{d,*}

\textsuperscript{a} Department of Mechanical Engineering, Taiyuan University of Science and Technology, Taiyuan 030024, China
\textsuperscript{b} Department of Computer Science and Technology, Taiyuan University of Science and Technology, Taiyuan 030024, China
\textsuperscript{c} Department of Computer Science and Control Engineering, North University of China, Taiyuan 030051, China
\textsuperscript{d} Department of Computer Science, University of Surrey, Guildford GU2 7XH, UK

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\section*{ABSTRACT}

Meta-heuristic algorithms, which require a large number of fitness evaluations before locating the global optimum, are often prevented from being applied to computationally expensive real-world problems where one fitness evaluation may take from minutes to hours, or even days. Although many surrogate-assisted meta-heuristic optimization algorithms have been proposed, most of them were developed for solving expensive problems up to 30 dimensions. In this paper, we propose a surrogate-assisted hierarchical particle swarm optimizer for high-dimensional problems consisting of a standard particle swarm optimization (PSO) algorithm and a social learning particle swarm optimization algorithm (SL-PSO), where the PSO and SL-PSO work together to explore and exploit the search space, and simultaneously enhance the global and local performance of the surrogate model. Our experimental results on seven benchmark functions of dimensions 30, 50 and 100 demonstrate that the proposed method is competitive compared with the state-of-the-art algorithms under a limited computational budget.

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\section*{1. Introduction}

For solving complex real-world optimization problems, computationally efficient algorithms are in high demand. Over the past decades, a variety of metaheuristic optimization methods, such as genetic algorithms, particle swarm optimization (PSO) algorithms and differential evolution algorithms have been proposed and successfully applied to many engineering optimization problems. Their success can partly be attributed to the fact that metaheuristic algorithms do not require that the objective functions be analytical and differentiable and have better global search capability. However, a large number of fitness evaluations are usually required for a metaheuristic algorithm to locate a near-optimal solution, which poses a grand challenge for them to be applied to computationally expensive problems, which are commonly seen in the real-world, such as structural optimization design of truss topology [42], aerodynamic optimization of airfoil shape [23], streamline optimization of vehicles [19], reliability optimization of complex systems [45], crashworthiness analysis of vehicles [43], and car engine management systems [39]. To address this challenge, surrogate models, also known as meta-models, are proposed to be used in lieu of the expensive performance evaluation to reduce the computational cost. Till now, a number of surrogate assisted metaheuristic optimization algorithms have been proposed and successfully applied in practice [10,13,14]. Models, widely been used as surrogates, include polynomial regression (PR) [20], radial basis function network (RBFN) [16,36], artificial neural network (ANN) [7,15,26], Kriging or Gaussian process (GP) [1,9,31] and support vector machines (SVM) [29,41]. Comparative studies have been conducted on the performance of modeling the problems with different fitness landscape...
characteristics [6, 12, 49], and the empirical results showed that the RBFN performs best for problems with different degrees of nonlinearity on small size of training data and scales relatively well with the increase in search dimension [12, 47], while the polynomial regression model is easy to be constructed and more convenient for analysis compared to other meta-models [12, 48]. As a class of statistical learning models, GP is well suited for capturing the global landscape of complex optimization problems, and can obtain results comparable with the RBFN and PR. Different from deterministic models, GP can provide an approximated fitness value together with a confidence level of the fitness approximation, which is particularly helpful in managing surrogates. However, optimizing the hyperparameters of the GP can become very time-consuming, which is a major impediment for GP to be widely employed, especially when the dimension of the decision space of the optimization problem is high. In this work, we decided to adopt the RBFN as the surrogate model due to its satisfactory performance on both low- and high-dimensional problems.

Over the past decades, various surrogate-assisted evolutionary algorithms have been reported in the literature. In general, existing frameworks can be divided into two categories according to the number of surrogates used, i.e., single-surrogate-assisted evolutionary algorithms and multi-surrogate-assisted ones. Jin et al. [15] adopted a neural network to be a global surrogate model to assist a covariance matrix adaptation evolution strategy and investigated the effectiveness of the individual-based and generation-based model management strategies. Praveen et al. [28] developed a surrogate-assisted particle swarm optimization by employing the radial-basis-function model to construct a global surrogate for prescreening the promising solutions to save the computational resource. In [30], Regis et al. proposed a framework for particle swarm optimization with an RBF global surrogate. They first generated multiple candidate solutions for each particle in each generation, then the surrogate was employed to select the promising positions to form the new population. Chugh et al. [4] proposed a Kriging-assisted reference vector guided evolutionary algorithm for optimization of computationally expensive many-objective optimization problems. This approach constructs a local Kriging model for each objective function, where the training samples were carefully selected for reducing the computation time by taking into consideration their relationships to the reference vectors. Liu et al. [22] adopted the Gaussian process model with the lower confidence bound to prescreen solutions in a differential evolution algorithm and a dimensional reduction technique was proposed to be utilized to enhance the accuracy of the GP model. The maximum dimension of the test problems used in [22] is 50 and the dimension is reduced to 4 before the surrogate is constructed. Moreover, Wang et al. [41] suggested a support vector regression (SVR) model assisted multi-objective evolutionary algorithm for proactive scheduling in the presence of stochastic machine breakdown and deterioration effect, where a SVR was employed in the evaluation of rescheduling cost to prescreen promising individuals to be reevaluated using the time-consuming simulation. A similar framework was also reported in [29] for appointment scheduling with uncertain examination time.

Multiple surrogates have been shown to perform better than single ones in assisting evolutionary algorithms. Zhou et al. [50] suggested to combine a global surrogate with a local surrogate in a hierarchical way to accelerate the evolutionary search, in which the main idea is using a global GP model to prescreen the offspring and an RBF based trust-region algorithm for local search. Sun et al. [36] introduced the global RBFN into the fitness inheritance based evolutionary framework [37] and proposed a two-layer surrogate based PSO for computationally expensive problems. Tenne et al. [40] developed an improved version of hierarchical surrogate-assisted memetic algorithm by using variable global and local RBF models. During the optimization, the optimal global and local RBF models were adaptively determined based on the leave-one-out cross validation. Tang et al. [38] presented a hybrid surrogate-assisted PSO, in which an RBF model constructed by interpolating the residual errors was added to a low order polynomial regression model to form the final hybrid surrogate model. Lim et al. [21] proposed a generalized framework for surrogate-assisted single- and multi-objective evolutionary optimization employing an ensemble-based local surrogate and a low-order polynomial based global surrogate in the local search. Inspired by ideas in active learning, Wang et al. [44] proposed an ensemble surrogate based model management method for surrogate-assisted PSO which searches for the promising and most uncertain candidate solutions to be evaluated using the expensive fitness function.

In most existing surrogate-assisted optimization algorithms using multiple surrogate models, the global surrogate model typically targets to smoothen out the local optima, while the local ones aim to capture the local details of the fitness function around the neighborhood of the current individuals. However, performing local search around each individual may be inefficient in helping find the global optimum and it might be more helpful to find the optimum of the search space that the population currently covers. Therefore, in this work, we propose a surrogate-assisted hierarchical PSO for high-dimensional bound-constrained optimization problems by combining SL-PSO with PSO, in which the SL-PSO algorithm aims to find the optimum of the current search region so that the surrogate is able to accurately learn the local fitness landscape around the optimum, whereas the PSO algorithm helps explore the search space gradually so that the surrogate can approximate the global profile of the fitness landscape. This way, the RBF surrogate is able to learn both the local details and the relatively global features of the fitness landscape as the search proceeds, enabling the search algorithms to find the optimum more efficiently. As the search carried out by SL-PSO is performed within each generation of the PSO, we term the proposed algorithm a surrogate-assisted hierarchical particle swarm optimization (SHPSO) algorithm.

The rest of the paper is organized as follows: Section 2 briefly reviews the two particle swarm optimization algorithms and the radial basis function network. The surrogate-assisted hierarchical particle swarm optimization is detailed in Section 3. Section 4 presents the experimental results with discussions. Section 5 concludes the paper with a summary and suggestions of future work.